K.C.S.E. MATHEMATICS PAPER 121/2 2003

SECTION I

Answer all the questions in this section

1. Solve the equation

$$\log_{10}(6x - 2) - 1 = \log_{10}(x - 3)$$ (3 marks)

2. Find the coordinates of the turning point of the curve whose equation is $y = 6 + 2x - 4x^2$. (3 marks)

3. Given that $a = \frac{1}{\sqrt{3}}$ and $b = \sqrt{13}$ express $2\sqrt{3} - 6\sqrt{39}$ in terms of a and b and simplify the answer. (3 marks)

4. a) Expand and simplify the binomial expression $(2 - x)^6$. (2 marks)

   b) Use the expansion up to the term in x^2 to estimate 1.99^6. (2 marks)

5. A mixed school can accommodate a maximum of 440 students. The number of girls must be at least 120 while the number of boys must exceed 150.

   Taking x to represent the number of boys and y the number of girls, write down all the inequalities representing the information above. (3 marks)

6. A colony of insects was found to have 250 insects at the beginning. Thereafter the number of insects doubled every 2 days.

   Find how many insects there were after 16 days. (3 marks)

7. The distances metres of an object varies partly with time t seconds and partly with the square root of time.

   Given that $s = 14$ when $t = 4$ and $s = 27$ when $t = 9$, write an equation connecting s and t. (4 marks)

8. Make c the subject of the formula:

$$T = x\sqrt{c^2 + d^2}$$ (3 marks)

9. A water pump costs Ksh 21 600 when new. At the end of first year its value depreciates by 25%. The depreciation at the end of the second year is 20% and thereafter the rate of depreciation is 15% yearly. Calculate the exact value of the water pump at the end of the fourth year. (3 marks)

10. Given that $x = 2i + j - 2k$, $y = -3i + 4j - k$ and $z = 5i + 3j + 2k$ and that $p = 3x - y + 2z$, find the magnitude of vector p to 3 significant figures (4 marks)

11. Solve the equation $3\tan^2 x - 4\tan x - 4 = 0$ for (4 marks)

12. There are three cars A, B, C in a race. A is twice as likely to win as B while B is twice as likely to win as C. Find the probability that

a) A wins the race (2 marks)

b) Either B and C wins the race (1 mark)

13. The velocity Vms^{-1} of a particle in motion is given by $V = 3t^2 - t + 4$, where t is time in seconds. Calculate the distance travelled by the particle between the time $t = 1$ second and $t = 5$ seconds. (3 marks)

SECTION II

Answer any **six** *questions from this section*

14. Given the simultaneous equations

$$5x + y = 19$$
$$-x + 3y = 0$$

a) Write the equations in matrix form. Hence solve simultaneous equations. (5 marks)

b) Find the distance of the point of intersection of the line $5x + y = 19$ and $-x + 3y = 9$ from the point (11, -2). (3 marks)

15. A dealer has three grades of coffee X, Y, and Z. Grade X costs sh 140 per kg, grade Y costs sh 160 per kg and grade Z costs sh 256 per kg.

a) The dealer mixes grade X and Y in the ratio 5 : 3 to make a brand of coffee which he sells at sh 180 per kg.

Calculate the percentage profit he makes. (3 marks)

b) The dealer makes a new brand by mixing the three grades of coffee, in the ratios X:Y=5:3 and Y:Z=2:5.

i) the ratio X:Y:Z in its simplest form (2 marks)

ii) the selling price of the new brand if he has to make a 30% profit. (3 marks)

16. A ship leaves P for port R through port Q. Q is 200km on a bearing of 220^0 from P. R is 420km on a bearing of 140^0 from Q.

a) Using the scale 1:4 000 000, draw a diagram showing the relative positions of the three ports P, Q and R. (3 marks)

b) By further drawing, on the same diagram, determine how far R is to the east of P. (2 marks)

c) If the ship had sailed directly from P to R at an average speed of 40 knots, find how long it would have taken to arrive at R. (Take 1 nautical mile = 1.853km) (3 marks)

17. Omondi makes two types of shoes: A and B. He takes 3 hours to make one pari of type A and 4 hours to make one pair of type B. He works for a maximum of 120 hours to make x pairs of type A and y pairs of type B. It costs him sh 400 to make a pair of type A and sh 150 to make a pair of type B.

His total cost does not exceed sh 9000. He must make 8 pairs of type A and more than 12 pairs of type B.

a) Write down four inequalities representing the information above. (2 marks)

b) On the grid provided, draw the inequalities and shade the unwanted regions. (3 marks)

c) Omondi makes a profit of sh 40 on each pair of type A and sh 70 on each pair of type B shoes.

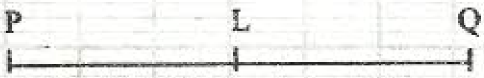
Use the graph in part (b) above to determine the maximum possible profit he makes. (2 marks)

18\. a) i) Find the coordinates of the stationary points on the curve $y = x^3 - 3x + 2$ (2 marks)

ii) For each stationary point determine whether it is minimum or maximum. (4 marks)

(b) In the space provided below, sketch the graph of the function $y = x^3 - 3x + 2$. (2 marks)

19\. The line PQ below, is 8cm long and L is its midpoint.



a) i) Draw the locus of point R above line PQ such that the are of triangle PQR is 12cm^2. (2 marks)

ii) Given that point R is equidistant from P and Q, show the position of point R. (2 marks)

b) Draw all the possible locii of a point T such that $\angle RQL = \angle RTL$ (4 marks)

20. a) Complete the table below, giving your values correct to 2 decimal places. (2 marks)

x	0^0	15^0	30^0	45^0	60^0	75^0	90^0	105^0	120^0	135^0	150^0	165^0	180^0
Cos x	1	0.87		0	-0.50	-1		0	0.50	0	0.50	0.87	1
Sin $(x+30^0)$	0.50	0.71	0.87	0.97	0.10		0.87	0.71	0.50		0		-0.50

b) Using the grid provided draw, on the same axes, the graph of $y = \cos 2x$ and $y = \sin(x + 30^0)$ for $0^0 \leq x \leq 180^0$.

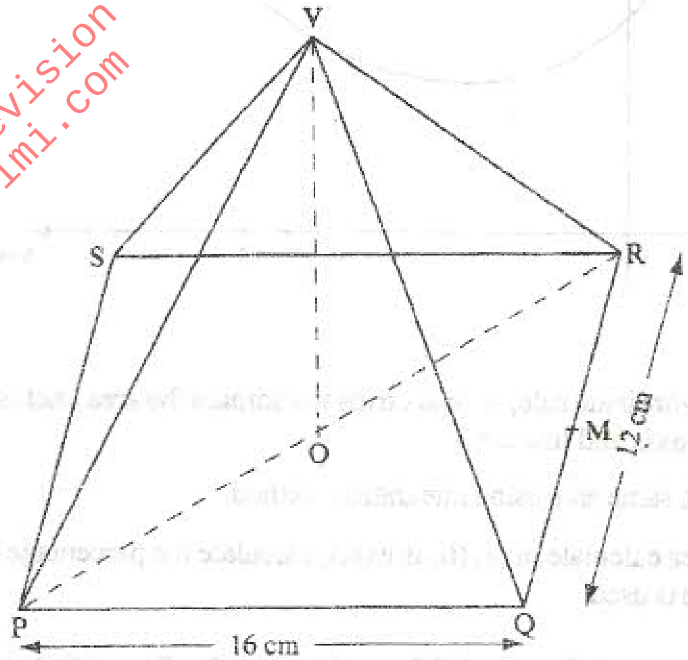
Take the scale: 1 cm 15^0 on the x-axis

4 cm for 1 unit on the y-axis. (4 marks)

c) Find the period of the curve y = cos 2x (1 mark)

d) Using the graphs in part *(b)* above, estimate the solutions to the equation
$\sin(x + 30^0) = \cos 2x$ (2 marks)

21. The figure below represents a right pyramid with vertex V and a rectangular base PQRS. VP = VQ = VR = S = 18cm. PQ = 16cm and QR = 12cm. M and O are the midpoints of QR and PR respectively.



Find:

a) the length of the projection of line VP on the plane PQRS (2 marks)

b) the size of the angle between line VP and the plane PQRS. (2 marks)

c) the size of the angle between the planes VQR and PQRS. (4 marks)

22. The masses of 40 babies in a certain clinic were recorded as follows:

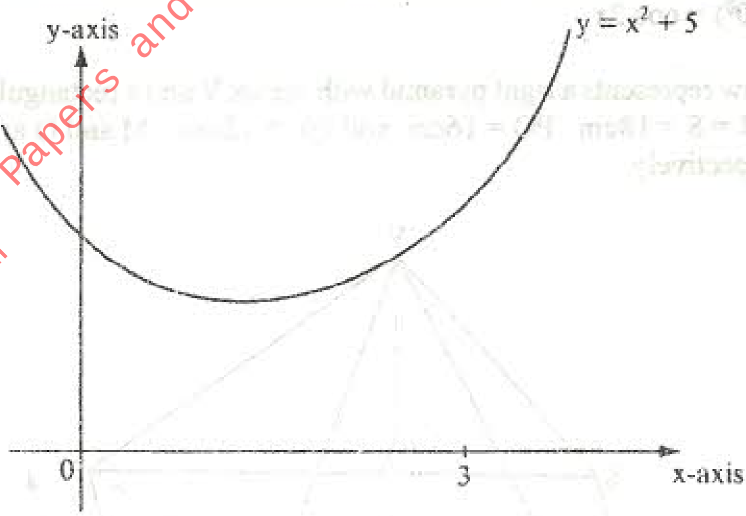
Mass in Kg	No. of babies
1.0 - 1.9	6
2.0 - 2.9	14
3.0 - 3.9	10
4.0 - 4.9	7
5.0 - 5.9	2
6.0 - 6.9	1

Calculate:

a) the interquartile range of the data (4 marks)

b) the standard deviation of the data using 3.45 as the assumed mean (4 marks)

23. The diagram below is a sketch of the curve $y = x^2 + 5$.

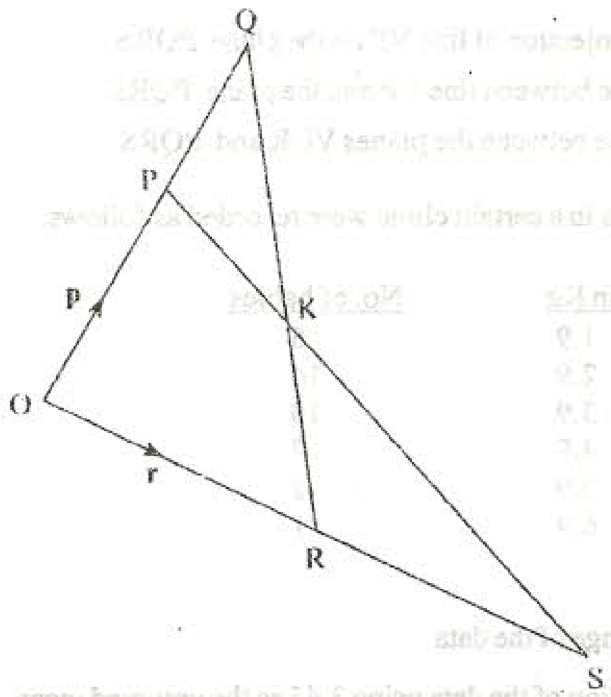


a) i) Use the mid-ordinate rule, with six trips to estimate the area enclosed by the curve, the x-axis the y-axis and line $x = 3$ (4 marks)

ii) Calculate the same area using integration method (2 marks)

b) Assuming the area calculate in (a) (ii) is exact, calculate the percentage error made when the mid-ordinate rule is used (2 marks)

24. In the figure below, vector OP = p and OR = r. Vector OS = 2r and OQ = $\frac{3}{2}$ p



a) Express in terms of P and r

i) QR (1 mark)

ii) PS (1 mark)

b) The line QR and PS intersect at K such that QK = mQR and PK = nPS, where m and n are scalars. Find two distinct expressions for OK in terms of P, r, m and n. Hence find the values of m and n. (5 marks)

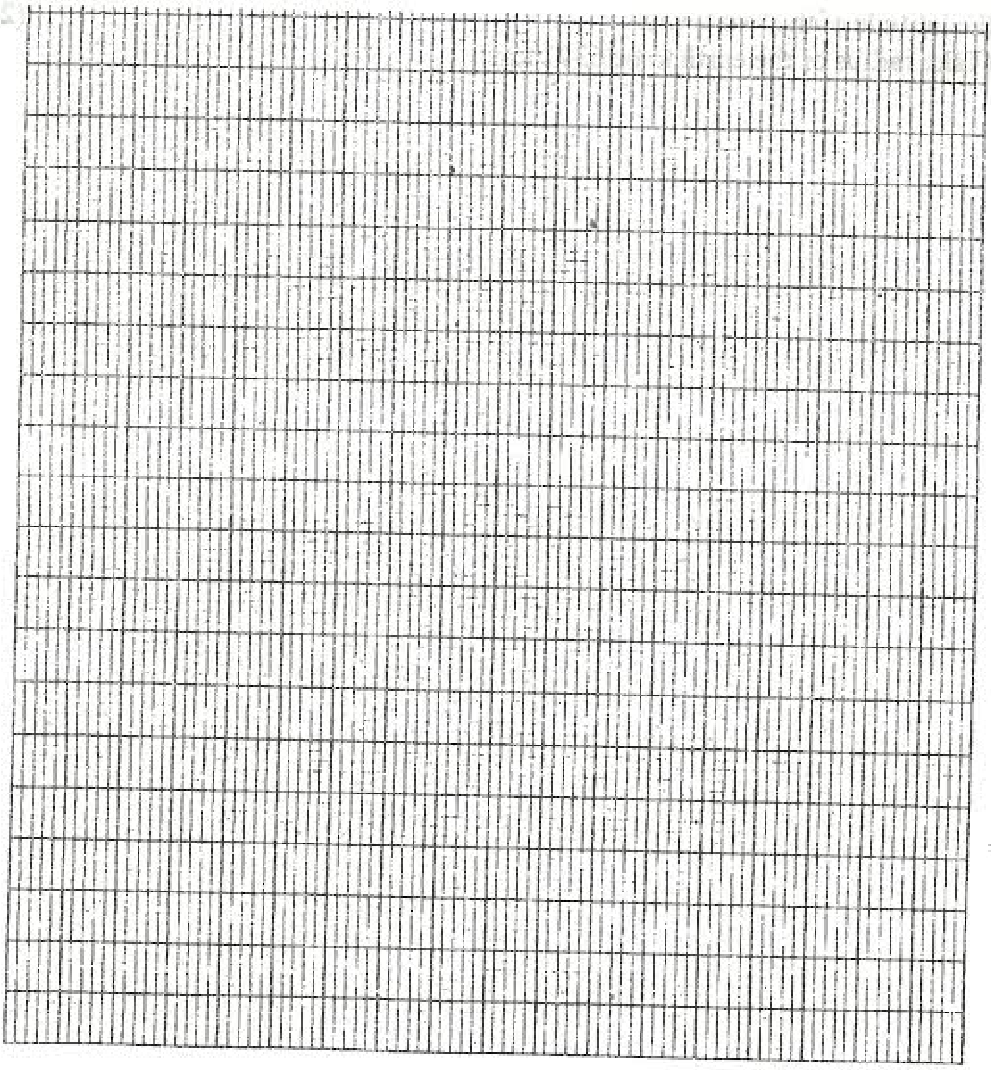
c) State the ratio PK : KS. (1 mark)

25. a) Complete the table below, for the function $y = 2x^2 + 4x - 3$ (2 marks)

x	-4	-3	-2	-1	0	1	2
$2x^2$	32		8	2	0	2	
$4x - 3$			-11		-3		5
y			-3			3	13

b) On the grid provided, draw the graph of the function $y = 2x^2 + 4x - 3$ for $-4 \leq x \leq 2$ and use the graph to estimate the roots of the equation $2x^2 + 4x - 3 = 0$ to 1 decimal place (3 marks)

c) In order to solve graphically the equation $2x^2 + x - 5 = 0$, a straight lien must be drawn to intersect the curve $y = 2x^2 + 4x - 3$.
Determine the equation of this straight line, draw the straight line and hence obtain the roots of the equation.
$2x^2 + x - 5 = 0$ to 1 decimal place (3 marks)



26. A businessman obtained a loan of sh 450 000 from a bank to buy a matatu valued at the same amount. The bank charges interest at 24% per annum compounded quarterly.

a) Calculate the total amount of money the businessman paid to clear the loan in 1½ years (3 marks)

b) The average income realised from the matatu per day was sh 1500. The matatu worked for 3 years at an average of 280 days per year.
Calculate the total income from the matatu (2 marks)

c) During the 3 yeas, the value of the matatu depreciated at the rate of 16% per annum. If the businessman sold the matatu at its new value, calculate the total profit he realised by the end of the 3 years. (3 marks)

27. Two town A and B lie on the same latitude in the northern hemisphere. When it is 8.00am at A, the time at B is 11.00am.

a) Given that the longitude of A is 15^{o}E, find the longitude of B (2 marks)

b) A plane leaves A for B and takes 3½ hours to arrive at B travelling along a parallel of latitude at 850km/h
Find:
i) the radius of the circle of latitude on which towns A and B lie (3 marks)
ii) the latitude of the two towns (2 marks)
(Take radius of the earth to be 6371 km)