THE KENYA NATIONAL EXAMINATIONS COUNCIL
Kenya Certificate of Secondary Education
Mathematics Paper 2
**2006**

Answer all the questions in this section.

1   In this question, show all the steps in your calculations, giving your answer at each stage. Use logarithms, correct to 4 decimal places, to evaluate

$$\sqrt[3]{\frac{36\cdot72\times(0\cdot46)^2}{185\cdot4}}$$

    (4 marks)

2   Make s the subject of the formula

$$\sqrt{p} = r\sqrt{1 - }$$

    (3 marks)

3   In the figure below R, T and S are points on a circle centre O. PQ is a tangent to the circle at T, POR is a straight line and $\angle QPR = 20°$.
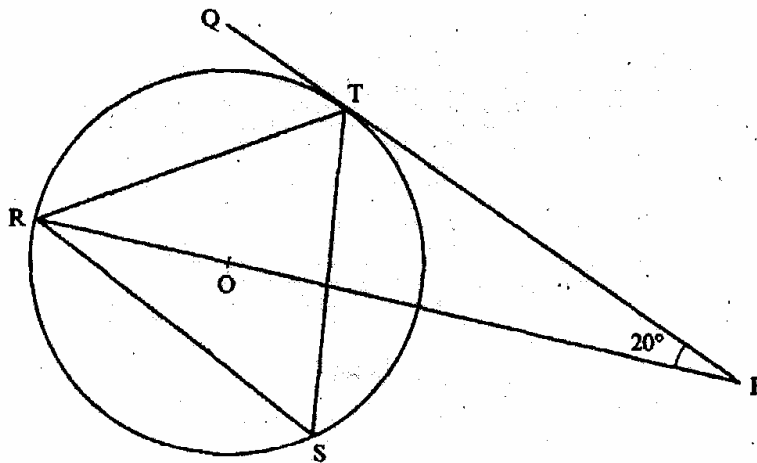

Find the size of $\angle RST$.   (2 marks)

By correcting each number to one significant figure, approximate the value of $788 \times 0\cdot006$. Hence calculate the percentage error arising from this approximation.   (3 marks)

5   The data below represents the ages in months at which 6 babies started walking:
9, 11, 12, 13, 11 and 10.
Without using a calculator, find the exact value of the variance of the data.

    (3 marks)

6   Without using a calculator or mathematical tables, simplify

$$\frac{3\sqrt{2} - \sqrt{3}}{2\sqrt{3} - \sqrt{2}}$$

    (3 marks)

7   The figure below shows a circle centre O and a point Q which is outside the circle.
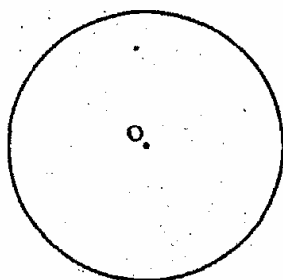
Using a ruler and a pair of compasses only, locate a point P on the circle such that angle OPQ = 90°.   (2 marks)

8   The table below is a part of tax table for monthly income for the year 2004.

Monthly taxable income in Ksh	Tax rate percentage (%) in each shilling
Under Ksh 9681	10%
From Ksh 9681 but under Ksh 18801	15%
From Ksh 18801 but under Ksh 27921	20%

In the year 2004, the tax on Kerubo's monthly income was Ksh 1916.

Calculate Kerubo's monthly income.   (3 marks)

9   Given that q i + ⅓ j + ⅔ k is a unit vector, find q.   (2 marks)

10   The points with coordinates (5, 5) and (⁻3, ⁻1) are the ends of a diameter of a circle centre A.
     Determine:
     (a) the coordinates of A   (1 mark)

     (b) the equation of the circle, expressing it in the form $x^2 + y^2 + ax + by + c = 0$
         where a, b and c are constants.   (3 marks)

11   Use binomial expression to evaluate

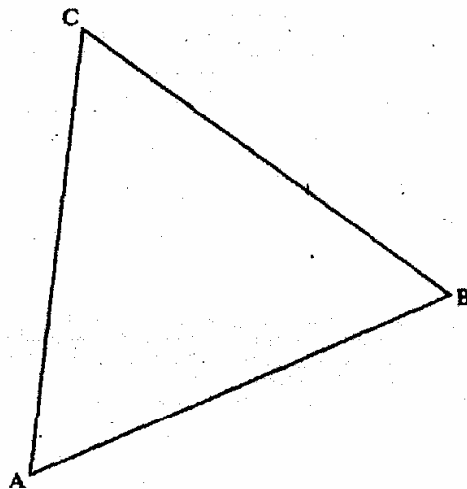$$\left(2 + \frac{1}{\sqrt{2}}\right)^5 + \left(2 - \frac{1}{\sqrt{2}}\right)^5$$

                                                                                                                                   (4 marks)

12   Three quantities t, x and y are such that t varies directly as x and inversely as the square root of y. Find the percentage decrease in t if x decreases by 4% when y increases by 44%.
                                                                                                                                   (4 marks)

13   The figure below is drawn to scale. It represents a field in the shape of an equilateral triangle of side 80 m.


The owner wants to plant some flowers in the field. The flowers must be at most, 60 m from A and nearer to B than to C. If no flower is to be more than 40 m from BC, show by shading, the exact region where the flowers may be planted.   (4 marks)

14   The table below shows some corresponding values of x and y for the curve represented by

$$y = \frac{1}{4}x^3 - 2.$$

x	-3	-2	-1	0	1	2	3
y	$-8\cdot8$	-4	$-2\cdot3$	-2	$-1\cdot8$	0	$4\cdot8$

On the grid provided below, draw the graph of $y = \frac{1}{4}x^3 - 2$ for $-3 \le x \le 3$. Use the graph to estimate the value of x when $y = 2$.   (3 marks)
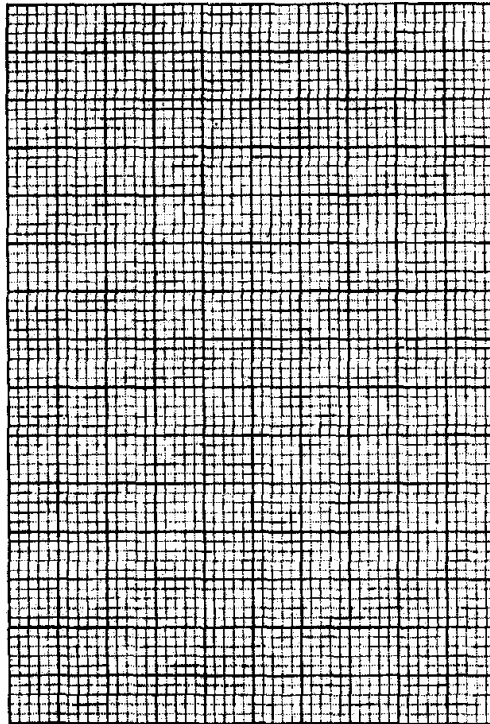
15 A particle moving in a straight line passes through a fixed point O with a velocity of 9 m/s. The acceleration of the particle, t seconds after passing through O is given by $a = (10 - 2t)$ m/s^2.

Find the velocity of the particle when $t = 3$ seconds. (3 marks)

16 Two places P and Q are at (36°N, 125°W) and (36°N, 55°E) respectively.

Calculate the distance in nautical miles between P and Q measured along the great circle through the North Pole. (3 marks)

SECTION II (50 marks)

Answer any five questions in this section.

17 (a) A certain sum of money is deposited in a bank that pays simple interest at a certain rate. After 3 years the total amount of money in the account is sh 358 400. The interest earned each year is sh 12 800.

Calculate:

(i) the amount of money which was deposited (2 marks)

(ii) the annual rate of interest that the bank paid. (2 marks)

(b) A computer whose marked price is sh 40 000 is sold at sh 56 000 on hire purchase terms.

(i) Kioko bought the computer on hire purchase terms. He paid a deposit of 25% of the hire purchase price and cleared the balance by equal monthly instalments of sh 2625. Calculate the number of instalments. (3 marks)

(ii) Had Kioko bought the computer on cash price terms he would have been allowed a discount of 12½% on marked price.

Calculate the difference between the cash price and hire purchase price and express it as a percentage of the cash price. (3 marks)

18 A garden measures 10 m long and 8 m wide. A path of uniform width is made all round the garden. The total area of the garden and the path is 168m².

(a) Find the width of the path.   (4 marks)

(b) The path is to be covered with square concrete slabs. Each corner of the path is covered with a slab whose side is equal to the width of the path.

The rest of the path is covered with slabs of side 50 cm. The cost of making each corner slab is sh 600 while the cost of making each smaller slab is sh 50.
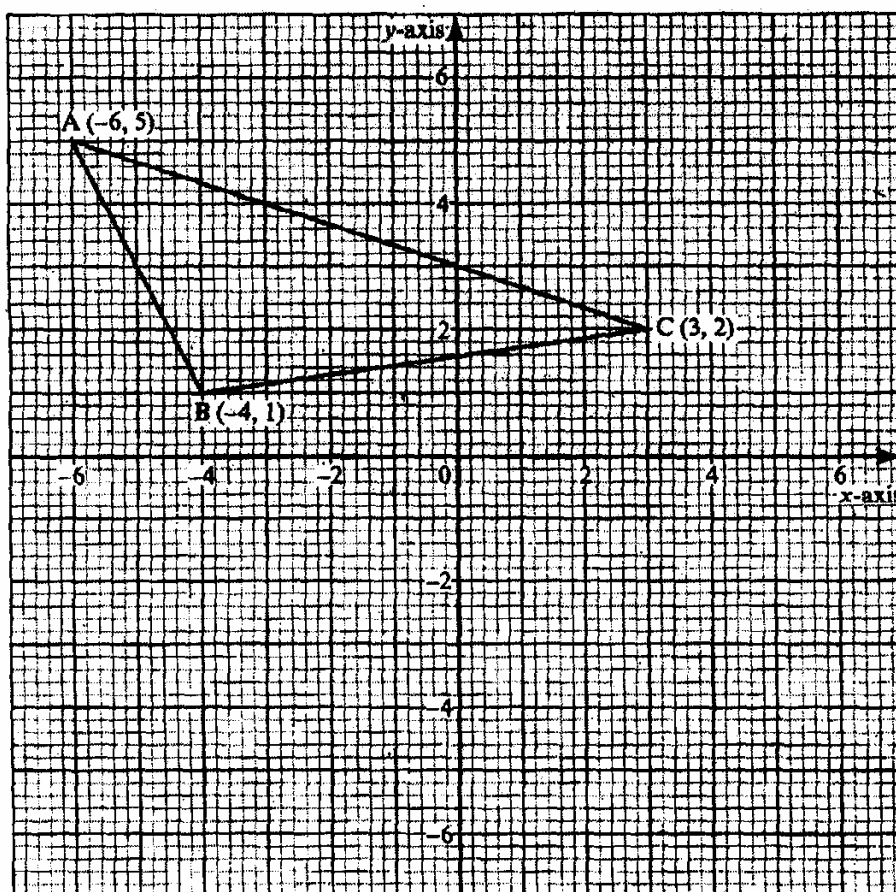
Calculate:

    (i) the number of the smaller slabs used   (3 marks)

    (ii) the total cost of the slabs used to cover the whole path.   (3 marks)

19 Triangle ABC is shown on the coordinate plane below.

(a) Given that A(-6, 5) is mapped onto A'(-6, -4) by a shear with y-axis invariant,
    (i) draw triangle A' B' C', the image of triangle ABC, under the shear.   (3 marks)
    (ii) determine the matrix representing the shear.   (2 marks)

(b) Triangle A′ B′ C′ is mapped onto A″ B″ C″ by a transformation defined by the

matrix $\begin{pmatrix} -1 & 0 \\ 1\frac{1}{2} & -1 \end{pmatrix}$

    (i) Draw triangle A″ B″ C″   (3 marks)

    (ii) Describe fully a single transformation that maps ABC onto A″ B″ C″.   (2 marks)

20   (a) Two integers, x and y, are selected at random from the integers 1 to 8. If the same integer may be selected twice, find the probability that:

    (i) $|x - y| = 2$ (2 marks)

    (ii) $|x - y|$ is 5 or more   (2 marks)

    (iii) $x > y$ (2 marks)

(b) A die is biased so that when tossed, the probability of a number r showing up, is given by P(r) = Kr where K is a constant and r = 1, 2, 3, 4, 5, 6 (the numbers on the faces of the die).

    (i) Find the value of K.   (2 marks)

    (ii) If the die is tossed twice, calculate the probability that the total score is 11.

                                                                    (2 marks)

21   A solution whose volume is 80 litres is made up of 40% of water and 60% of alcohol. When x litres of water are added, the percentage of alcohol drops to 40%.

(a) Find the value of x.   (4 marks)

(b) Thirty litres of water is added to the new solution. Calculate the percentage of alcohol in the resulting solution.   (2 marks)

(c) If 5 litres of the solution in (b) above is added to 2 litres of the original solution, calculate in the simplest form, the ratio of water to that of alcohol in the resulting solution.
                                                                 (4 marks)

22   The product of the first three terms of a geometric progression is 64.

If the first term is a, and the common ratio is r,

(a) Express r in terms of a.   (3 marks)

(b) Given that the sum of the three terms is 14
    (i) Find the values of a and r and hence write down two possible sequences each up to the 4th term.   (5 marks)
    (ii) Find the product of the 50th terms of the two sequences.   (2 marks)

23 Mwanjoki Flying Company operates a flying service. It has two types of aeroplanes. The smaller one uses 180 litres of fuel per hour while the bigger one uses 300 litres per hour.

The fuel available per week is 18 000 litres. The company is allowed 80 flying hours per week. To keep the aeroplane in good condition the bigger aeroplane must be flown for x hours per week while the smaller aeroplane must be flown for y hours per week.

(a) Write down all the inequalities representing the above information. (3 marks)

(b) On the grid provided on page 21, draw all the inequalities in (a) above by shading the unwanted regions. (4 marks)

(c) The profit on the smaller aeroplane is sh 4 000 per hour while that on the bigger one is sh 6 000 per hour.

Use the graph drawn in (b) above to determine the maximum profit that the company made per week. (3 marks)
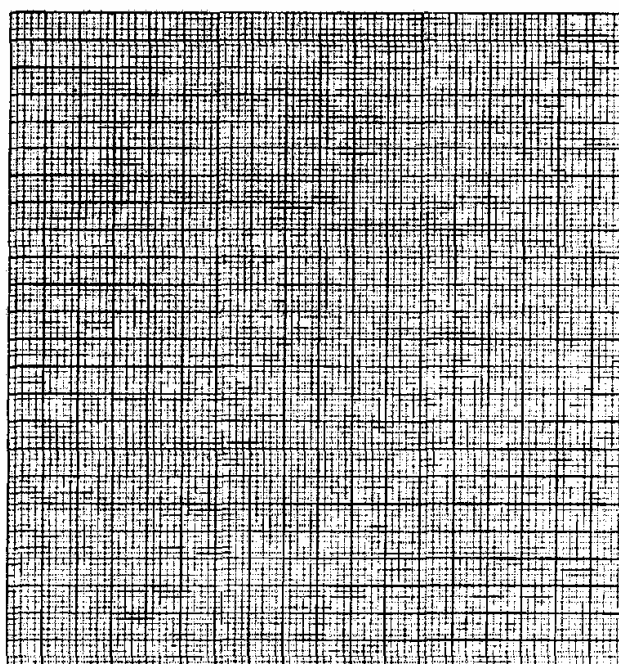
24 The diagram below shows a sketch of the line $y = 3x$ and the curve $y = 4 - x^2$ intersecting at points P and Q.
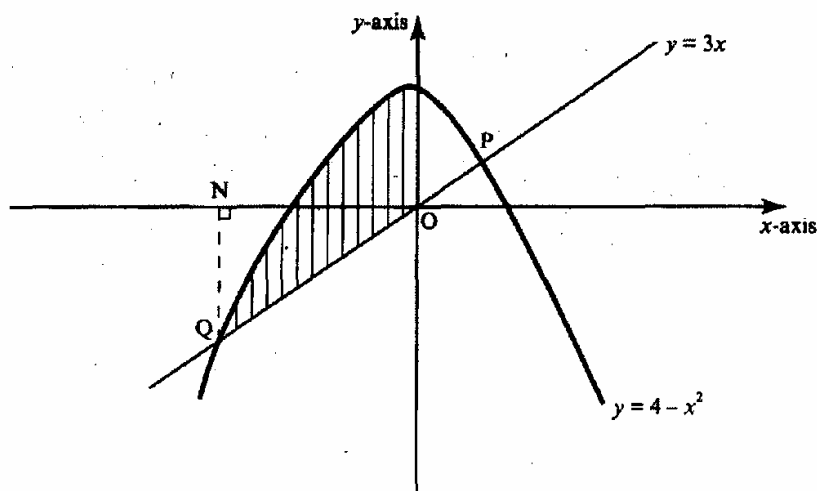
(*a*) Find the coordinates of P and Q. (4 marks)

(*b*) Given that QN is perpendicular to the *x*-axis at N, calculate
   (i) the area bounded by the curve $y = 4 - x^2$, the *x*-axis and the line QN. (2 marks)

   (ii) the area of the shaded region that lies below the *x*-axis. (2 marks)

   (iii) the area of the region enclosed by the curve $y = 4 - x^2$, the line $y = 3x$ and the *y*-axis. (2 marks)