K.C.S.E. MATHEMATICS PAPER 121/2 2000

SECTION I

Answer all the questions in this section.

1. The distance from a fixed point of a particle in motion at any time \( t \) seconds is given by
   \[ s = t^3 - \frac{5}{2} t^2 + 2t + 5 \text{ metres} \]
   Find its:
   a) acceleration after 1 seconds
   b) velocity when acceleration is zero.
   (1 mark) (2 marks)

2. Solve the equation \( 2 \sin^2(x - 30^\circ) = \cos 60^\circ \) for \(-180^\circ \leq x \leq 180^\circ\)
   (3 marks)

3. Three representatives are to be selected randomly from a group of 7 girls and 8 boys.
   Calculate the probability of selecting two girls and one boy.
   (3 marks)

4. Expand \((1 + x)^3\), hence, use the expansion to estimate \((1.04)^3\) correct to 4 decimal places.
   (4 marks)

5. In the figure below, BT is a tangent to the circle at B. AXCT and BXD are straight lines.
   \( AX = 6 \text{ cm}, CT = 8 \text{ cm}, BX = 4.8 \text{ cm} \) and \( XD = 5 \text{ cm} \).
   
   ![](image)
   
   Find the length of
   a) \( XC \)
   b) \( BT \)
   (2 marks) (2 marks)

6. Make \( x \) the subject of the formula
   \[ p = \left( \frac{xy}{z + x} \right)^{\frac{1}{3}} \]
   (4 marks)

7. Find the value of \( x \) that satisfies the equation
   \[ \log (x + 5) = \log 4 - \log (x + 2) \]
   (3 marks)

8. Given \( \sin \theta = \frac{3}{4} \) and \( \theta \) is an acute angle find:
   a) \( \tan \theta \), giving your answer in surd form
   b) \( \sec \theta \)
9. The length and breadth of a rectangular paper were measured to the nearest centimetre and found to be 18 cm and 12 cm respectively. Find the percentage error in its perimeter (3 marks)

10. A tailor intends to buy a sewing machine which costs Ksh.48,000. He borrows the money from a bank. The loan has to be repaid at the end of the second year. The bank charges an interest at the rate of 24% per annum compounded half-yearly. Calculate the total amount payable to the bank (4 marks)

11. On the figure below lines ABC and DC are tangents to the circle at B and D respectively. \(\angle ACD=40^\circ\) and \(\angle ABE=60^\circ\).

Giving reasons find the size of:

a) \(\angle CBD\)

b) \(\angle CDE\)

12. The acceleration \(a\) m/s\(^2\) of a particle moving in a straight line is given by \(a = 18t - 4\), where \(t\) is time in seconds. The initial velocity of the particle is 2 m/s

a) Find the expression for velocity in terms of \(t\) (2 marks)

b) Determine the time when the velocity is again 2 m/s (1 mark)
SECTION II

13. The eleventh term of an arithmetic progression is four times its second term. The sum of the first seven terms of the same progression is 175.
   a) Find the first term and the common difference of the progression. (4 marks)
   b) Given that the $p^{th}$ term of the progression is greater than 124, find the least value of $p$. (4 marks)

14. The charge, $C$ shillings per person for a certain seminar is partly fixed and partly inversely proportional to the total number $N$ of people.
   a) Write down an expression for $C$ in terms of $N$. (1 mark)
   b) When 100 people attended the charge is sh. 8,700 per person while for 35 people the charge is sh. 10,000 per person. (1 mark)
   c) If a person had paid the full amount and does not attend, the fixed charge is refunded. A group of people paid but ten per cent of them did not attend. After the refund the organiser remained with sh. 574,000. Find the number of people initially in the group. (4 marks)

15. The curve of the equation $y = 2x + 3x^2$, has $x = -\frac{2}{3}$ and $x = 0$ and $x$-intercepts. The area bounded by the $x$-axis $x = -\frac{2}{3}$ and $x = 2$ is shown by the sketch below.

Find:
   a) $\int (2x + 3x^2) \, dx$ (2 marks)
   b) the area bounded by the curve, $x$-axis, $x = -\frac{2}{3}$ and $x = 2$ (6 marks)

16. The line segment BC given below is one side of triangle ABC.
   a) Use a ruler and compasses to complete the construction of triangle ABC in which $\angle ABC = 45^\circ$, AC = 5.6cm and angle BAC is obtuse. (2 marks)
   b) Draw the locus of a point $P$ such that $P$ is equidistant from a point $O$ and passes through the vertices of triangle ABC. (2 marks)
   c) Locate point $D$ on the locus of $P$ equidistant from lines $BC$ and $BO$. $Q$ lies in the region enclosed by lines $BD$, $BO$ extended and the locus of $P$. Shade the locus of $Q$. (4 marks)

17. A theatre has a seating capacity of 250 people. The charges are sh. 100 for an ordinary seat and sh. 160 for a special seat. It costs sh. 16,000 to stage a show and the theatre must make a

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18. A triangular plot ABC is such that AB = 36 m, BC = 40 m and AC = 42 m
   a) Calculate the:
      i) Area of the plot in square metres
      ii) Acute angle between the edges AB and BC  
          (3 marks)
   b) A water tap is to be installed inside the plot such that the tap is equidistant from each of the vertices A, B and C. Calculate the distance of the tap from vertex A.  
          (2 marks)

19. In a Form 1 class there are 22 girls and 18 boys. The probability of a girl completing the secondary education course is \( \frac{3}{4} \) whereas that of a boy is \( \frac{1}{2} \).
   a) A student is picked at random from the class. Find the possibility that,
      i) The student picked is a boy and will complete the course.
ii) The student picked will complete the course (2 marks)

b) Two students are picked at random. Find the probability that they are a boy and a girl and that both will not complete the course

20. a) Complete the table below for the equation 
\[ y = 2x^2 + 5x^2 - x - 6 \] 

<table>
<thead>
<tr>
<th>x</th>
<th>-4</th>
<th>-3</th>
<th>-2</th>
<th>-1</th>
<th>0</th>
<th>1</th>
<th>2</th>
</tr>
</thead>
<tbody>
<tr>
<td>x^2</td>
<td>-128</td>
<td>-54</td>
<td></td>
<td>0</td>
<td>2</td>
<td>16</td>
<td></td>
</tr>
<tr>
<td>5x^2</td>
<td>80</td>
<td>45</td>
<td>20</td>
<td>5</td>
<td>0</td>
<td>5</td>
<td>20</td>
</tr>
<tr>
<td>-x</td>
<td>-4</td>
<td>-3</td>
<td>-2</td>
<td>-1</td>
<td>0</td>
<td>5</td>
<td>-1</td>
</tr>
<tr>
<td>-6</td>
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<td>-6</td>
</tr>
<tr>
<td>y</td>
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<td>-6</td>
<td></td>
<td>0</td>
<td>5</td>
<td>-6</td>
<td>0</td>
</tr>
</tbody>
</table>

(2 marks)

b) On the grid provided draw the graph
Use 2cm to represent 1 unit on x-axis and 1 cm to represent 5 units on the y-axis (3 marks)

c) By drawing a suitable line use the graph in (b) to solve the equation, 
\[ 2x^3 + 5x^2 + x - 4 = 0 \] 

(3 marks)
21. The figure below shows triangle OAB in which M divides OA in the ratio 2:3 and N divides OB in the ratio 4:1. AN and BM intersect at X.

a) Given that OA = a and OB = b, express in terms of a and b:
   i) AN
   ii) BM

b) If AX = sAN and BX = tBM, where s and t are constants, write two expressions for OX in terms of a, b, s and t.
   Find the value of s

Hence write OX in terms of a and b.

22. A plane leaves an airport A (38.5°N, 37.05°W) and flies due North to a point B on latitude 52°N.
   a) Find the distance covered by the place (4 marks)
   b) The plane then flies due East to a point C, 2400km from B. Determine the position of C
      (Take the value of \( \pi \) as \( \frac{22}{7} \) and radius of the earth as 6370km)

23. Matrix \( P \) is given by \[
\begin{pmatrix}
4 & 7 \\
5 & 8
\end{pmatrix}
\]
   a) Find \( P^{-1} \) (2 marks)
   b) Two institutions, Elimu and Somo, purchase beans at sh. \( b \) per bag and maize at sh. \( m \) per bag. Elimu purchased 8 bags of beans and 14 bags of maize for sh 47,600. Somo purchased 10 bags of beans and 16 bags of maize for sh 57,400.
      i) Form a matrix equation to represent the information above (1 mark)
      ii) Use the matrix \( P^{-1} \) to find the prices of one bag of each item (3 marks)
   c) The price of beans later went up by 5% and that of maize remained constant. Elimu bought the same quantity of beans but spent the same total amount of money as before on the two items. State the new ratio of beans to maize. (2 marks)
24. a) Complete the table for the equation
\[ y = 2\sin(3x + 30^\circ) \]  
(2 marks)

<table>
<thead>
<tr>
<th>x</th>
<th>0°</th>
<th>10°</th>
<th>20°</th>
<th>30°</th>
<th>40°</th>
<th>50°</th>
<th>60°</th>
<th>70°</th>
<th>80°</th>
<th>90°</th>
</tr>
</thead>
<tbody>
<tr>
<td>3x + 30°</td>
<td>30°</td>
<td>60°</td>
<td>90°</td>
<td>120°</td>
<td>150°</td>
<td>180°</td>
<td>210°</td>
<td>240°</td>
<td>270°</td>
<td>300°</td>
</tr>
<tr>
<td>y = 2\sin(3x + 30°)</td>
<td>1</td>
<td>1.73</td>
<td>2</td>
<td>0</td>
<td>-2</td>
<td>-1.73</td>
<td>-2</td>
<td>-1.73</td>
<td>-2</td>
<td>-1.73</td>
</tr>
</tbody>
</table>

b) Using the grid provided, draw the graph of \( y = 2\sin(3x + 30^\circ) \) for \( 0^\circ \leq x \leq 90^\circ \).  
Take 1 cm to represent 50 on the x-axis and 2 cm to represent 1 unit on the y-axis  
(3 marks)

c) Use the graph in (b) to find the range of \( x \) that satisfy the inequality \( y \geq 1.6 \)