MATHEMATICS PAPER 1
2011

SECTION 1 (50 marks)

Answer all the questions in this section in the spaces provided.

1. Without using a calculator, evaluate;

\[
\frac{2\frac{1}{5} + \frac{2}{3} \text{ of } 3\frac{3}{4} - 4\frac{1}{6}}{1\frac{1}{4} - 2\frac{2}{5} \div 1\frac{1}{3} + 3\frac{3}{4}}
\]

(3 marks)
2. The diagonal of a rectangular garden measures $11\frac{1}{4}$ m while its width measures $6\frac{3}{4}$ m. Calculate the perimeter of the garden. (2 marks)

3. A motorist took 2 hours to travel from one town to another town and 1 hour 40 minutes to travel back. Calculate the percentage change in the speed of the motorist. (3 marks)

4. A square room is covered by a number of whole rectangular slabs of sides 60 cm by 42 cm. Calculate the least possible area of the room in square metres. (3 marks)

5. Given that $\sin (x + 60)^\circ = \cos (2x)^\circ$, find $\tan (x + 60)^\circ$. (3 marks)
6. Simplify the expression:

\[
\frac{4x - 9x^3}{3x^2 - 4x - 4}
\]

7. The external length, width and height of an open rectangular container are 41 cm, 21 cm and 15.5 cm respectively. The thickness of the material making the container is 5 mm. If the container has 8 litres of water, calculate the internal height above the water level.

(4 marks)

8. Factorise \(2x^2y^2 - 5xy - 12\)

(2 marks)

9. Using a ruler and a pair of compases only:
(a) construct a parallelogram PQRS in which PQ = 6 cm, QR = 4 cm and angle SPQ = 75°;  
(3 marks)

(b) determine the perpendicular distance between PQ and SR.  
(1 mark)

10. The masses of people during a clinic session were recorded as shown in the table below.

<table>
<thead>
<tr>
<th>Mass (kg)</th>
<th>40-44</th>
<th>45-49</th>
<th>50-54</th>
<th>55-59</th>
<th>60-64</th>
<th>65-69</th>
<th>70-74</th>
</tr>
</thead>
<tbody>
<tr>
<td>No. of</td>
<td>1</td>
<td>2</td>
<td>12</td>
<td>10</td>
<td>2</td>
<td>2</td>
<td>1</td>
</tr>
</tbody>
</table>

Calculate the mean mass.  
(3 marks)
11. A customer paid Ksh. 5 880 for a suit after she was allowed a discount of 2% on the selling price. If the discount had not been allowed, the shopkeeper would have made a profit of 20% on the sale of the suit. Calculate the price at which the shopkeeper bought the suit.

(3 marks)

12. Three vertices of a parallelogram PQRS are P(-1, 2), Q(8, -5) and R (5,0).

(a) On the grid provided below draw the parallelogram PQRS. (1 mark)
(b) Determine the length of the diagonal QS. (2 marks)

13. In January, Mambo donated $\frac{1}{6}$th of his salary to a children's home while Simba donated $\frac{1}{5}$th of his salary to the same children's home. Their total donation for January was Ksh. 14 820. In February, Mambo donated $\frac{1}{8}$th of his salary to the children's home while Simba donated $\frac{1}{12}$th of his salary to the children's home. Their total donation for February was Ksh 8 675. Calculate Mambo's monthly salary. (4 marks)
14. (a) Express 10500 in terms of its prime factors. (1 mark)

(b) Determine the smallest positive number P such that 10500P is a perfect cube. (2 marks)

15. Three police posts X, Y and Z are such that Y is 50 km on a bearing of 060° from X while Z is 70 km from Y and on a bearing of 300° from X.

(a) Using a suitable scale, draw a diagram to represent the above situation. (3 marks)

(b) Determine the distance, in km, of Z from X. (1 mark)
16. A small cone of height 8 cm is cut off from a bigger cone to
leave a frustum of height 16 cm, if the volume of the smaller
cone is 160 cm³, find the volume of the frustum.

(SECTION II) (50 marks)

Answer any five questions in this section in the spaces provided.

17. A solid consists of a cone and a hemisphere. The common
diameter of the cone and the hemisphere is 12 cm and the
slanting height of the cone is 10 cm. (a) Calculate correct to
two decimal places:

(i) the surface area of the solid;

(ii) the volume of the solid.
(b) If the density of the material used to make the solid is 1.3 g/cm³, calculate its mass in kilograms. (3 marks)

18. Makau made a journey of 700 km partly by train and partly by bus. He started his journey at 8.00 a.m. by train which travelled at 50 km/h. After alighting from the train, he took a lunch break of 30 minutes. He then continued his journey by bus which travelled at 75 km/h. The whole journey took $11\frac{1}{2}$ hours.

(a) Determine:

(i) the distance travelled by bus; (4 marks)

(ii) the time Makau started travelling by bus. (3 marks)
(b) The bus developed a puncture after travelling $187\frac{1}{2}$ km. It took 15 minutes to replace the wheel. Find the time taken to complete the remaining part of the journey (3 marks)

19. (a) The product of the matrices

\[
\begin{pmatrix}
0 & 1 \\
2 & p
\end{pmatrix}
\text{ and }
\begin{pmatrix}
5 & 0.5 \\
p & p - 2
\end{pmatrix}
\]

is a singular matrix.

Find the value of p. (3 marks)

(b) A saleswoman earned a fixed salary of Ksh $x$ and a commission of Ksh $y$ for each item sold. In a certain month she sold 30 items and earned a total of Ksh 50,000. The following month she sold 40 items and earned a total of Ksh 56,000.

(i) Form two equations in $x$ and $y$. (2 marks)

(ii) Solve the equations in (i) above using matrix method. (3 marks)
(iii) In the third month she earned Ksh 68 000. Find the number of items sold.

(2 marks)

20. In a triangle ABC, BC = 8 cm, AC = 12 cm and angle ABC = 120°.

(a) Calculate the length of AB, correct to one decimal place. (4 marks)

(b) If BC is the base of the triangle, calculate, correct to one decimal place:

(i) the perpendicular height of the triangle; (2 marks)
(ii) the area of the triangle; (2 marks)

(iii) the size of angle ACB. (2 marks)

21. (a) Using the trapezium rule with seven ordinates, estimate the area of the region
bounded by the curve \( y = -x^2 + 6x + 1 \), the lines \( x = 0 \), \( y = 0 \)
and \( x = 6 \). (5 marks)

(b) Calculate:

(i) the area of the region in (a) above by integration; (3 marks)
(iii) the percentage error of the estimated area to the actual area of the region, correct to two decimal places. (2 marks)

22. The displacement, s metres, of a moving particle after t seconds is given by,

\[ s = 2t^3 - 5t^2 + 4t + 2. \]  

Determine:

(a) the velocity of the particle when \( t = 3 \) seconds; (3 marks)

(b) the value of \( t \) when the particle is momentarily at rest; (3 marks)

(c) the displacement when the particle is momentarily at rest; (2 marks)

(d) the acceleration of the particle when \( t = 3 \) seconds. (2 marks)
23. In the figure below, ABCD is a trapezium, AB is parallel to DC, diagonals AC and DB intersect at X and DC = 2 AB. \( AB = a, \ DA = d, \ AX = k \ AC \) and \( DX = hDB \), where \( h \) and \( k \) are constants.
(a) Find in terms of \( a \) and \( d \):

(i) \( BC \); 

(ii) \( AX \);

(iii) \( DX \);

(2 marks)

(2 marks)

(1 marks)

(b) Determine the values of \( h \) and \( k \)

(5 marks)

24. The frequency table below shows the daily wages paid to casual workers by a certain company.

<table>
<thead>
<tr>
<th>Wages in shillings</th>
<th>100-150</th>
<th>150-200</th>
<th>200-300</th>
<th>300-400</th>
<th>400-600</th>
</tr>
</thead>
<tbody>
<tr>
<td>No: of workers</td>
<td>160</td>
<td>120</td>
<td>380</td>
<td>240</td>
<td>100</td>
</tr>
</tbody>
</table>

(a) Draw a histogram to represent the above information.

(5 marks)
(b) (i) State the class in which the median wage lies. (1 mark)

(ii) Draw a vertical line, in the histogram, showing where the median wage lies. (1 mark)

(c) Using the histogram, determine the number of workers who earn sh 450 or less per day. (3 marks)
1. Use logarithms, correct to 4 decimal places, to evaluate
\[ \sqrt[3]{\frac{83.46 \times 0.0054}{1.56^2}} \]
(4 marks)

2. Three grades A, B, and C of rice were mixed in the ratio 3:4:5. The cost per kg of each of the grades A, B and C were Ksh 120, Ksh 90 and Ksh 60 respectively.

Calculate:
(a) The cost of one kg of the mixture; (2 marks)
(b) The selling price of 5 kg of the mixture given that the mixture was sold at 8% profit,
3. Make s the subject of the formula.

\[ \frac{w}{y} = \sqrt{s + \frac{r}{s}} \]  

(3 marks)

4. (a) Solve the inequalities \(2x - 5 > -11\) and \(3 + 2x \leq 13\), giving the answer as a combined inequality. (3 marks)

(b) List the integral values of \(x\) that satisfy the combined inequality in (a) above.  

(1 mark)

5. In the figure below, ABCD is a cyclic quadrilateral. Point O is the centre of the circle. Angle ABO = 30° and angle ADO = 40°.

![Diagram of a cyclic quadrilateral with points A, B, C, D, and O, where O is the centre of the circle, and angles ABO and ADO are marked as 30° and 40° respectively.]

Calculate the size of angle BCD.  

(2 marks)
6. The ages in years of five boys are 7, 8, 9, 10 and 11 while those of five girls are 4, 5, 6, 7 and 8. A boy and a girl are picked at random and the sum of their ages is recorded.

(a) Draw a probability space to show all the possible outcomes. (1 mark)

(b) Find the probability that the sum of their ages is at least 17 years. (1 mark)

7. The vertices of a triangle are A(1,2), B(3,5) and C(4,1). The coordinates of C' the image of C under a translation vector T, are (6-2).

(a) Determine the translation vector T. (1 mark)

(b) Find the coordinates of A' and B' under translation vector T. (2 marks)

8. Write sin 45° in the form $\frac{1}{\sqrt{a}}$ where $a$ is a positive integer. Hence simplify $\frac{\sqrt{8}}{1 + \sin 45°}$, leaving the answer in surd form. (3 marks)
9. The radius of a spherical ball is measured as 7 cm, correct to the nearest centimeter. Determine, to 2 decimal places, the percentage error in calculating the surface area of the ball.

(4 marks)

10. (a) In the figure below, lines NA and NB represent tangents to a circle at points A and B.
Use a pair of compasses and ruler only to construct the circle. (2 marks)

(b) Measure the radius of the circle. (1 mark)

11. Expand and simplify the expression.

\[
\left(a + \frac{1}{2}\right)^4 + \left(a - \frac{1}{2}\right)^4
\]

(3 marks)

12. The figure below represents a scale drawing of a rectangular piece of land, RSTU.

RS = 9 cm and ST = 7 cm.
13. An electric post P, is to be erected inside the piece of land. On the scale drawing, shade the possible region in which P would lie such that PU > PT and PS ≤ 7 cm.

(3 marks)

Vector $\mathbf{OP} = 6\mathbf{i} - \mathbf{j}$ and $\mathbf{OQ} = -2\mathbf{i} - 5\mathbf{j}$. A point N divides PQ internally in the ratio 3:1.

Find $\mathbf{PN}$ in terms of $\mathbf{i}$ and $\mathbf{j}$. (3 marks)

14. A point M ($60^\circ$N, $18^\circ$E) is on the surface of the earth. Another point N is situated at a distance of 630 nautical miles east of M.

Find:
(a) the longitude difference between M and N; (2 marks)

(b) The position of N. (1 mark)
15. The equation of a circle centre \((a, b)\) is \(x^2 - y^2 - 6x - 10y + 30 = 0\). Find the values of \(a\) and \(b\). (3 marks)

16. The table below shows values of \(x\) and \(y\) for the function \(y = 2 \sin 3x^\circ\) in the range

<table>
<thead>
<tr>
<th>(x^\circ)</th>
<th>0</th>
<th>15</th>
<th>30</th>
<th>45</th>
<th>60</th>
<th>75</th>
<th>90</th>
<th>105</th>
<th>120</th>
<th>135</th>
<th>150</th>
</tr>
</thead>
<tbody>
<tr>
<td>(y)</td>
<td>0</td>
<td>1.4</td>
<td>2</td>
<td>1.4</td>
<td>0</td>
<td>-1.4</td>
<td>-2</td>
<td>-1.4</td>
<td>0</td>
<td>1.4</td>
<td>2</td>
</tr>
</tbody>
</table>

(a) On the grid provided, draw the graph of \(y = 2 \sin 3x\). (2 marks)

(b) From the graph determine the period. (1 mark)
SECTION II (50 marks)

Answer only five questions in this section in the spaces provided.

17. The cash price of a laptop was Ksh 60 000. On hire purchase terms, a deposit of Ksh 7 500 was paid followed by 11 monthly installments of Ksh 6 000 each.

(a) Calculate:
(i) the cost of a laptop on hire purchase terms; (2 marks)

(ii) the percentage increase of hire purchase price compared to the cash price. (2 marks)

(b) An institution was offered a 5% discount when purchasing 25 such laptops on cash terms. Calculate the amount of money paid by the institution. (2 marks)

(c) Two other institutions, X and Y, bought 25 such laptops each. Institutions X bought the laptops on hire purchase terms. Institution Y bought the laptops on cash terms with no discount by securing a loan from a bank. The bank charged 12% p.a. compound interest for two years. Calculate how much more money institution Y paid than institution X. (4 marks)

18. The first, fifth and seventh terms of an Arithmetic Progression (AP) correspond to the first three consecutive terms of a decreasing
Geometric Progression (G.P). The first term of each progression is 64, the common difference of the AP is $d$ and the common ratio of the G.P is $r$.

(a) (i) Write two equations involving $d$ and $r$. (2 marks)

(ii) Find the values of $d$ and $r$. (4 marks)

(b) Find the sum of the first 10 terms of:

(i) The Arithmetic Progression (A.P); (2 marks)

(ii) The Geometric Progression (G.P). (2 marks)
The vertices of a rectangle are A(-1,-1), B(-4,-1), C(-4,-3) and D(-1,-3).

(a) On the grid provided, draw the rectangle and its image A'B'C'D' under a transformation whose matrix is

\[
\begin{pmatrix}
-2 & 0 \\
0 & -2
\end{pmatrix}
\]

(4 marks)

b) A'' B'' C'' D'' is the image of A' B' C' D' under a transformation matrix,
(i) Determine the coordinates of A", B", C" and D". (2 marks)

(ii) On the same grid draw the quadrilateral A" B" C" D". (1 mark)

(c) Find the area of A" B" C" D". (3 marks)

20. A parent has two children whose age difference is 5 years. Twice the sum of the ages of the two children is equal to the age of the parent.

(a) Taking \( x \) to be the age of the elder child, write an expression for:

(i) the age of the younger child; (1 mark)

(ii) the age of the parent. (1 mark)

(b) In twenty years time, the product of the children's ages will be 15 times the age of their parent.
(i) Form an equation in \( x \) and hence determine the present possible ages of the elder child. (4 marks)

(ii) Find the present possible ages of the parent. (2 marks)

(iii) Determine the possible ages of the younger child in 20 years time. (2 marks)

21. The table below shows values of \( x \) and some values of \( y \) for the curve \( y = x^3 + 2x^2 - 3x - 4 \) for \(-3 \leq x \leq 2\).

<table>
<thead>
<tr>
<th>( x )</th>
<th>-3</th>
<th>-2.5</th>
<th>-2</th>
<th>-1.5</th>
<th>-1</th>
<th>-0.5</th>
<th>0</th>
<th>0.5</th>
<th>1</th>
<th>1.5</th>
<th>2</th>
</tr>
</thead>
<tbody>
<tr>
<td>( y )</td>
<td>-4.0</td>
<td>-0.4</td>
<td>1.6</td>
<td>0</td>
<td>-4.0</td>
<td>^ .9</td>
<td>6</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

(a) Complete the table by filling in the missing values of \( y \), correct to 1 decimal place. (2 marks)

(b) On the grid provided, draw the graph of \( y = x^3 + 2x^2 - 3x - 4 \).

Use the scale: 1 cm represents 0.5 units on \( x \)-axis.
1 cm represents 1 unit on \( y \)-axis. (3 marks)
(c) Use the graph to:

(i) solve the equation $x^3 + 2x^2 - 3x - 4 = 0$; (3 marks)

(ii) estimate the coordinates of the turning points of the curve. (2 marks)
The figure below represents a rectangular based pyramid $VABCD$. $AB = 12$ cm and $AD = 16$ cm. Point $O$ is vertically below $V$ and $VA = 26$ cm.

Calculate:
(a) the height, $VO$, of the pyramid; (4 marks)

(b) the angle between the edge $VA$ and the plane $ABCD$; (3 marks)
(c) the angle between the planes VAB and ABCD. (3 marks)

23 The cost $C$, of producing $n$ items varies partly as $n$ and partly as the inverse of $n$. To produce two items it costs Ksh 135 and to produce three items it costs Ksh 140. Find:

(a) the constants of proportionality and hence write the equation connecting $C$ and $n$; (5 marks)

(b) the cost of producing 10 items; (2 marks)

(c) the number of items produced at a cost of Ksh 756. (3 marks)
24. A building contractor has two lorries, P and Q, used to transport at least 42 tonnes of sand to a building site. Lorry P carries 4 tonnes of sand per trip while lorry Q carries 6 tonnes of sand per trip. Lorry P uses 2 litres of fuel per trip while lorry Q uses 4 litres of fuel per trip. The two lorries are to use less than 32 litres of fuel. The number of trips made by lorry P should be less than 3 times the number of trips made by lorry Q. Lorry P should make more than 4 trips.

(a) Taking $x$ to represent the number of trips made by lorry P and $y$ to represent the number of trips made by lorry Q, write the inequalities that represent the above information.  

\[ \begin{align*} 
4x + 6y & \geq 42, \\
2x + 4y & < 32, \\
x & < 3y, \\
x & > 4. 
\end{align*} \]

(4 marks)

(b) On the grid provided, draw the inequalities and shade the unwanted regions.  

(4 marks)

(c) Use the graph drawn in (b) above to determine the number of trips made by lorry P and by lorry Q to deliver the greatest amount of sand.  

(2 marks)