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## 121/2

MATHEMATICS ALT. A

## PAPER 2

JUNE 2016
TIME: $\mathbf{2}^{1 ⁄ 2}$ hours

## THE 4MCK JOINT EXAMINATION

Kenya Certificate of Secondary Education (KCSE)
MATHEMATICS ALT. A

## Paper 2

$21 / 2$ hours

## Instructions to Candidates

(a) Write your name and admission number in the spaces provided above.
(b) This Paper consists of Two sections. Section I and section II.
(c) Answer ALL the questions in section I and ONLY five questions from section II.
(d) Show all the steps in your calculations, giving your answers at each stage in the spaces provided below each question.
(e) Marks may be given for correct working even if the answer is wrong..
(f) Non-programmable silent electronic calculators and KNEC Mathematical table may be used, except where stated otherwise.
(g) Candidates should check the question paper to ascertain that all the pages are printed ad indicated, and that no questions are missing.

## FOR EXAMINER'S USE ONLY

## Section I

| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | Total |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |

## Section II

| 17 | 18 | 19 | 20 | 21 | 22 | 23 | 24 | Total |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
|  |  |  |  |  |  |  |  |  |

Grand
Total


## SECTION I: (50 MARKS)

## Answer all the questions in this section

1. The base and height of a triangle were measured to be 10 cm and 4.0 cm respectively. Calculate the percentage error in calculating its area.
2. Make $y$ the subject of the formula

$$
P=d\left(\frac{a+y^{2}}{a-y^{2}}\right)^{1 / 2}
$$

(3 marks)
3. a) Simplify:

: express it in the form
$a+b \sqrt{ }$; where $a, b$, and $c$ are rational numbers.
(2 marks)
b) Hence find the values of $a, b$ and $c$.
(1 mark)
4. In the figure below (not drawn to scale), points $\mathrm{A}, \mathrm{B}, \mathrm{C}$ and D lie on the circumference of the circle and PAQ is a tangent of the circle at A. Given that angles BAQ and ABD are $40^{\circ}$ and $30^{\circ}$ respectively; and $\mathrm{CB}=\mathrm{CD}$;


P
A
Q

## Determine angle;

a) PAD
(1 mark)
b) BCD
(1 mark)
c) CDA
(1 mark)
5. a) A rectangle whose area is 6 square units undergoes a transformation represented by matrix $M=\left(\begin{array}{cc}-3 & 2 \\ -1 & 2\end{array}\right)$. Determine the area of its image. $\quad$ (2 marks)
b) A point P undergoes a transformation by matrix M above in (a) and gets mapped onto $\mathrm{P}^{\prime}(6,4)$.

Find the co-ordinates of P .
(2 marks)
6. The position of points $A$ and $B$ are $\mathbf{a}=4 \mathbf{i}+4 \mathbf{j}-6 \mathbf{k}$ and $\mathbf{b}=10 \mathbf{i}+4 \mathbf{j}+12 \mathbf{k}$ respectively. A point D divides AB in ratio 2:1. Find the magnitude of the position vector of D .
(4 marks)
7. An arc of length 24 cm subtends an angle of $1 \cdot 6$ radians at the center of the circle. Calculate the area of the circle. (Take $\pi=3 \cdot 142$ ). (3 marks)
8. Form a quadratic equation whose roots are $-1 / 4$ and $2 / 3$ expressing your answer in the form $\mathrm{ax}^{2}+\mathrm{bx}+\mathrm{c}=0$ where $\mathrm{a}, \mathrm{b}$, and c are integers.
9. Solve the equation for $0^{\circ} \leq \mathrm{x} \leq 180^{\circ}$

$$
-3 \operatorname{Cos}(2 x+10)^{\circ}=0 \cdot 6
$$

10. a) Expand and simplify the first four terms of the binomial expression $(2-1 / 4 x)^{6}$
(2 marks)
b) Use your expansion in (a) above to evaluate (1.975) ${ }^{6}$ correct to 2 decimal places. (2 marks)
11. A curve whose gradient function is $3 x^{2}-4 x+5$ passes through point $(-1,1)$. Determine the equation of the curve.
12. Without using tables or calculator, find the value of $x$ if;

$$
\log (20 x+35)-\log 2 x=1+2 \log 3
$$

13. The probability of Joyce passing her exams is $2 / 3$. If she fails in her exams, she repeats and the probability of her passing increases by $10 \%$. Find the probability she will pass her exams after repeating.
14. Tangent PQ and chords AB and CD intersect at point Q outside the circle in the figure below. If $\mathrm{AB}=5 \mathrm{~cm}, \mathrm{DQ}=4 \mathrm{~cm}$ and $\mathrm{BQ}=6 \mathrm{~cm}$;


Determine;
a) The length of chord CD
b) The length of tangent PQ
15. Find the co-ordinates of the center and the diameter of the circle whose equation is

$$
\begin{equation*}
2 x^{2}+2 y^{2}+16 x-4 y=-2 \tag{3marks}
\end{equation*}
$$

16. A right pyramid $A B C D V$ below (not drawn to scale) has a square base $A B C D$ whose $A B$ $=10 \mathrm{~cm}$ and its vertex V is such that $\mathrm{AV}=\mathrm{BV}=\mathrm{CV}=\mathrm{DV}=13 \mathrm{~cm}$


Calculate the angle between planes BCV and ABCD .

## SECTION II: (50 MARKS): Answer any five questions only in this section.

17. The table below shows the income tax rates:

| Monthly Income (K£) | Rate (\%) |
| :---: | ---: |
| $1-2000$ | 10 |
| $2001-4000$ | 15 |
| $4001-6000$ | 20 |

Above 600030
Mr. Mutua earns a basic salary of Kshs 64,000 per month. He is entitled to allowances amounting to Kshs 45,600 per month; and a personal relief of Kshs 1056 per month. Calculate
a) His total taxable income in $\mathrm{K} £$ p.m (2 marks)
b) The total P.A.Y.E tax he pays in Kshs p.m
(8 marks)
18. The figure below shows a scale diagram of a rectangular piece of land ABCD in which $A B=10 \mathrm{~cm}$ and $A D=6 \mathrm{~cm}$.

a) Using a pair of compasses and a ruler only; construct on the diagram above
i) The locus of a point P equidistant from lines AB and AD
(2 marks)
ii) The locus of a point Q such that $\mathrm{AQ}=6 \mathrm{~cm}$ (2 marks)
iii) The locus of a point R such that $\angle \mathrm{ARB}=90^{\circ}$
(2 marks)
b) A bore hole H lies inside the piece of land ABCD above such that its nearer to AD than AB , and $\mathrm{AH} \leq 6 \mathrm{~cm}$ and also angle AHB greater than $90^{\circ}$. Show by shading out the unwanted regions, and labeling H on the diagram above, the region where the borehole H lies.
(4 marks)
19. A ship sails non stop from port $\mathrm{P}\left(20^{\circ} \mathrm{N}, 30^{\circ} \mathrm{E}\right)$ northwards to a port $\mathrm{Q}\left(45^{\circ} \mathrm{N}, 30^{\circ} \mathrm{E}\right)$ then due west to port R a distance of 4500 nm
a) Find the total distance in nautical miles travelled by the ship.
(2 marks)
b) Determine the position of port R , to nearest degree.
c) Find the average speed of the ship in knots if it took 8 hours.
d) Determine the local time at $R$ when the ship arrived there if it set off at $P$ at 9:00am local time.
20. The sketch below shows the graphs of curve $y=4 x-x^{2}$ and line $y=x$, which intersect at points O and B

a) Determine the coordinates of points $\mathrm{O}, \mathrm{A}, \mathrm{B}$ and C .
(5marks)
b) Use integration to calculate the area of the region enclosed by the curve $y=4 x-x^{2}$ and the line $\mathrm{y}=\mathrm{x}$ (shaded)
21. a) The $n^{\text {th }}$ term of a sequence is given by $T n=n^{2}-n+3$

Determine:
i) the $10^{\text {th }}$ term of the sequence
(1 mark)
ii) the difference between $10^{\text {th }}$ and $25^{\text {th }}$ terms of the sequence
(2 marks)
iii) the value of $n$ if $T n=243$
(3 marks)
c) In a research station, it was found that the number of bacteria doubles in every one hour. If the number of bacteria started with 200; how long does it take for the bacteria to lit 1 million.
22. The marks obtained by 10 students in a maths test were; $25,24,22,23, x, 26,21,23,22$,and 27. The sum of the squares of the marks $\left(\Sigma \mathrm{x}^{2}\right)$ is 5154 .
a) Calculate
i) The value of $x$
(2 marks)
ii) The mean
(2 marks)
iii) The standard deviation
(4 marks)
b) If each mark was increased by 3 and the doubled; determine i) The new mean
(2 marks)
ii) The new standard deviation
(2 marks)
23. The volume $\mathrm{Vcm}^{3}$ of a solid varies jointly as the square of the radius rcm of its base and its height hcm . Given that $\mathrm{V}=180 \mathrm{~cm}^{3}$, when $\mathrm{r}=3 \mathrm{~cm}$ and $\mathrm{h}=10 \mathrm{~cm}$;
a) Determine the value of constant of proportionality
(2 marks)
b) Find the diameter of the base when $\mathrm{V}=480 \mathrm{~cm}^{3}$ and $\mathrm{h}=15 \mathrm{~cm}$
(3 marks)
c) Calculate the percentage change in volume V when r is increased in ratio $5: 4$ and h is decreased by $5 \%$. Correct to 4 significant figures.
24. A tailoring company makes two types of garments A and B. Garment A requires 3 meters of material to make while B requires 2 meters of material. The company uses not more than 600 meters of material in making the garments. It must make at most 100 of type A and at least 80 of type B.
a) By letting the number of type A garments made be $x$ and type B garment made be $y$; form four inequalities for the above information.
b) Represent the inequalities in a (a) above in the squared grid provided.
(4 marks)
c) If the company makes a profit of Kshs 100 on garment A and 60 on garment B ; find the number of garments of each type that should be made to maximize the profit. Hence determine the maximum profit.


