

**CIFA PART I SECTION 1**  
**FINANCIAL MATHEMATICS**

**MONDAY: 23 November 2020.**

**Time Allowed: 3 hours.**

**Answer ALL questions. Marks allocated to each question are shown at the end of the question. Show ALL your workings.**

**QUESTION ONE**

- (a) Suggest four applications of financial mathematics in finance and investments. (4 marks)
- (b) The following data shows the distribution of profits realised by 80 branches of a multinational company for the year ended 31 December 2019:

Profit/loss Sh. "million"	Number of branches
- 20 < - 10	4
- 10 < 0	8
0 < 10	14
10 < 20	38
20 < 30	10
30 < 40	6

**Required:**

- (i) The average profit of the multinational company. (2 marks)
- (ii) The standard deviation and the coefficient of variation. (4 marks)
- (iii) The median profit. (3 marks)
- (iv) One of the managers commented that the distribution of profit seems to be negatively skewed. Explain to him the meaning of "negatively skewed profit". (2 marks)
- (c) Jackson Kipruto deposited Sh.100,000 in two different banks, X and Y, dividing the amount into two investments. Bank X calculates interest at a compound interest rate of 7% per annum while bank Y calculates interest at the rate of 6% per annum convertible semi-annually. At the end of 3 years, Jackson received Sh.21,264.70 as the return on his investment.

**Required:**

- The amount deposited by Jackson in bank X and bank Y respectively. (5 marks)
- (Total: 20 marks)**

**QUESTION TWO**

- (a) Examine three benefits of investing in corporate bonds. (6 marks)
- (b) James Kimondo deposited Sh.252,350 in an investment fund at an annual interest rate of 5% compounded continuously.

**Required:**

- (i) The compound amount and the interest earned after 8 years. (4 marks)
- (ii) The effective rate of interest. (2 marks)
- (iii) The time required for the original amount of Sh.252,350 to grow to Sh.1,030,000. (2 marks)



- (c) Fredrick Nabuye's family bought a house for Sh.28,325,000. They made a down payment of Sh.5,665,000 and took a 30-year mortgage for Sh.22,660,000 at an annual interest rate of 6%.

**Required:**

- (i) The amount of the monthly payment required to amortise the loan. (2 marks)
- (ii) The total amount of interest paid when the loan is amortised over the 30 year period. (2 marks)
- (iii) The part of the first payment that is interest and the part that is applied to reducing the loan amount. (2 marks)

**(Total: 20 marks)**

**QUESTION THREE**

- (a) As a junior financial analyst at Bob capital, you have been approached by a client with a newspaper advertisement on treasury bonds from Central Bank. The client is confused on various terminologies used in the article and is seeking your guidance.

**Required:**

Explain to your client the following terms:

- (i) Coupon rate. (1 mark)
- (ii) Call date. (1 mark)
- (iii) Redemption date. (1 mark)
- (iv) Yield to maturity. (1 mark)
- (v) Discount bond. (1 mark)
- (vi) Premium bond. (1 mark)

- (b) The following data relates to all the employees of ABC Ltd.:

<u>Age (Years)</u>	<u>Bachelors Degree</u>	<u>Masters Degree</u>	<u>Total</u>
Under 30	90	10	100
30 – 40	20	30	50
Over 40	40	10	50
<b>Total</b>	<b>150</b>	<b>50</b>	<b>200</b>

**Required:**

- (i) A joint probability table to represent the above data. (4 marks)
- (ii) The probability that an employee selected at random from the company has only a bachelor's degree. (2 marks)
- (iii) The probability that an employee selected at random has a masters degree given that he is over 40 years. (2 marks)
- (iv) The probability that an employee selected at random is under 30 years given that he has only a bachelors degree. (2 marks)

- (c) XYZ Ltd. intends to invest in a project with the following estimated cash flows:

<b>Year</b>	<b>0</b>	<b>1</b>	<b>2</b>	<b>3</b>	<b>4</b>	<b>5</b>	<b>6</b>
<b>Cash in flows (Sh."000")</b>	-	-	30,000	40,000	40,000	40,000	40,000
<b>Cash out flows (Sh."000")</b>	120,000	20,000	-	-	-	-	-

The company's cost of capital is 12% per annum.

**Required:**

- (i) The net present value (NPV) of the project. (3 marks)
- (ii) Advise the management of XYZ Ltd. on whether to accept or reject the project based on your results in (c) (i) above. (1 mark)

**(Total: 20 marks)**



**QUESTION FOUR**

(a) Describe four applications of differential calculus in finance. (4 marks)

(b) Hedmax Limited has developed the following functions to assist in monitoring the operations of the company:

$$\begin{aligned} \text{Total cost (TC), } C &= 14 + 3x \\ \text{Total revenue (TR), } R &= 19x - 2x^2 \end{aligned}$$

Where X is the level of output in thousands of shillings (Sh. "000").

**Required:**

- (i) The total profit function. (2 marks)
- (ii) The breakeven point. (3 marks)
- (iii) The level of demand necessary to maximize profit. (2 marks)
- (iv) The maximum profit obtained. (1 mark)
- (c) Christopher Marwa bought 12 dozen digital watches for his shop located at the Nairobi Central Business District. He sold some of the digital watches at Sh.400 each and some at Sh.250 each. His total receipts amount to Sh.51,000.

**Required:**

The number of watches he sold at Sh.400 and Sh.250 respectively. (4 marks)

(d) In a certain profession, it was found that 10,000 employees had a mean monthly income of Sh.175,000 with a standard deviation of Sh.5,000. The income in this profession follows a normal distribution.

**Required:**

The percentage of the employees earning a monthly income greater than Sh.166,800. (4 marks)

**(Total: 20 marks)**

**QUESTION FIVE**

(a) Assess two properties of coefficient of correlation. (4 marks)

(b) The table below shows the dividend yields (x) and share prices (y) of a number of companies listed on the Securities Exchange during the year 2019:

Company	Dividend yield (%)	Share price (Sh.)
A	12	130
B	16	145
C	17	167
D	18	187
E	22	220
F	24	234

**Required:**

- (i) The product moment correlation coefficient. (6 marks)
- (ii) Interpret your result in (b) (i) above. (2 marks)
- (c) The prices per unit of the items forming the consumption bundle of an average family in two periods and the percentage of total family budget allocated to these items are as shown in the table below:

Expenditure Item	Price in period 1 (Sh.)	Price in period 2 (Sh.)	Percentage expenditure (%)
Food	30,000	34,800	40
Rent	10,000	12,000	20
Clothing	20,000	25,000	15
Fuel	4,000	5,000	15
Miscellaneous	12,000	18,000	10

**Required:**

- (i) An appropriate consumer price index number using the above data. (5 marks)
- (ii) Comment on your result in (c) (i) above. (1 mark)
- (iii) Explain how time reversal test could be used in choosing an appropriate index. (2 marks)

**(Total: 20 marks)**



Present Value Interest factor of 1 Received at the End of  $n$  Periods at  $r$  Percent:

$$PVIF_{r,n} = 1 / (1+r)^n = (1+r)^{-n}$$

Period	1%	2%	3%	4%	5%	6%	7%	8%	9%	10%	11%	12%	13%	14%	15%	16%	20%	24%	25%	30%
1	0.9901	0.9804	0.9709	0.9615	0.9524	0.9434	0.9346	0.9259	0.9174	0.9091	0.9009	0.8929	0.8850	0.8772	0.8696	0.8621	0.8333	0.8065	0.8000	0.7692
2	0.9803	0.9612	0.9426	0.9246	0.9070	0.8900	0.8734	0.8573	0.8417	0.8264	0.8116	0.7972	0.7831	0.7695	0.7561	0.7432	0.6944	0.6504	0.6400	0.5917
3	0.9706	0.9423	0.9151	0.8890	0.8638	0.8396	0.8163	0.7938	0.7722	0.7513	0.7312	0.7118	0.6931	0.6750	0.6575	0.6407	0.5787	0.5245	0.5120	0.4552
4	0.9610	0.9238	0.8885	0.8548	0.8227	0.7921	0.7629	0.7350	0.7084	0.6830	0.6587	0.6355	0.6133	0.5921	0.5718	0.5523	0.4823	0.4230	0.4096	0.3501
5	0.9515	0.9057	0.8626	0.8219	0.7835	0.7473	0.7130	0.6806	0.6499	0.6209	0.5935	0.5674	0.5428	0.5194	0.4972	0.4761	0.4019	0.3411	0.3277	0.2693
6	0.9420	0.8880	0.8375	0.7903	0.7462	0.7050	0.6663	0.6302	0.5963	0.5645	0.5346	0.5066	0.4803	0.4556	0.4323	0.4104	0.3349	0.2751	0.2621	0.2072
7	0.9327	0.8706	0.8131	0.7599	0.7107	0.6651	0.6227	0.5835	0.5470	0.5132	0.4817	0.4523	0.4251	0.3996	0.3759	0.3538	0.2791	0.2218	0.2097	0.1594
8	0.9235	0.8535	0.7894	0.7307	0.6768	0.6274	0.5820	0.5403	0.5019	0.4665	0.4339	0.4039	0.3762	0.3506	0.3269	0.3050	0.2326	0.1789	0.1678	0.1226
9	0.9143	0.8368	0.7664	0.7026	0.6446	0.5919	0.5439	0.5002	0.4604	0.4241	0.3909	0.3606	0.3329	0.3075	0.2843	0.2630	0.1938	0.1443	0.1342	0.0943
10	0.9053	0.8203	0.7441	0.6756	0.6139	0.5584	0.5083	0.4632	0.4224	0.3855	0.3522	0.3220	0.2946	0.2697	0.2472	0.2267	0.1615	0.1164	0.1074	0.0725
11	0.8963	0.8043	0.7224	0.6496	0.5847	0.5268	0.4751	0.4289	0.3875	0.3505	0.3173	0.2875	0.2607	0.2366	0.2149	0.1954	0.1346	0.0938	0.0859	0.0558
12	0.8874	0.7885	0.7014	0.6246	0.5568	0.4970	0.4440	0.3971	0.3555	0.3186	0.2858	0.2567	0.2307	0.2076	0.1869	0.1685	0.1122	0.0757	0.0687	0.0429
13	0.8787	0.7730	0.6810	0.6006	0.5303	0.4688	0.4150	0.3677	0.3262	0.2897	0.2575	0.2292	0.2042	0.1821	0.1625	0.1452	0.0935	0.0610	0.0550	0.0330
14	0.8700	0.7579	0.6611	0.5765	0.5051	0.4423	0.3878	0.3405	0.2992	0.2633	0.2320	0.2046	0.1807	0.1597	0.1413	0.1252	0.0779	0.0492	0.0440	0.0254
15	0.8613	0.7430	0.6419	0.5553	0.4810	0.4173	0.3624	0.3152	0.2745	0.2394	0.2090	0.1827	0.1599	0.1401	0.1229	0.1079	0.0649	0.0397	0.0352	0.0195
16	0.8528	0.7284	0.6232	0.5339	0.4581	0.3936	0.3387	0.2919	0.2519	0.2176	0.1883	0.1631	0.1415	0.1229	0.1069	0.0930	0.0541	0.0320	0.0281	0.0150
17	0.8444	0.7142	0.6050	0.5134	0.4363	0.3714	0.3166	0.2703	0.2311	0.1978	0.1696	0.1456	0.1252	0.1078	0.0925	0.0802	0.0451	0.0258	0.0225	0.0116
18	0.8360	0.7002	0.5874	0.4936	0.4155	0.3503	0.2959	0.2502	0.2120	0.1799	0.1528	0.1300	0.1108	0.0946	0.0808	0.0691	0.0376	0.0208	0.0180	0.0089
19	0.8277	0.6864	0.5703	0.4746	0.3957	0.3305	0.2765	0.2317	0.1945	0.1635	0.1377	0.1161	0.0981	0.0829	0.0703	0.0596	0.0313	0.0168	0.0144	0.0068
20	0.8195	0.6730	0.5537	0.4564	0.3769	0.3118	0.2584	0.2145	0.1784	0.1486	0.1240	0.1037	0.0868	0.0728	0.0611	0.0514	0.0261	0.0135	0.0115	0.0053
21	0.8114	0.6598	0.5375	0.4388	0.3589	0.2942	0.2415	0.1987	0.1637	0.1351	0.1117	0.0926	0.0768	0.0638	0.0531	0.0443	0.0217	0.0109	0.0092	0.0040
22	0.8034	0.6468	0.5219	0.4220	0.3418	0.2775	0.2257	0.1839	0.1502	0.1228	0.1007	0.0826	0.0688	0.0569	0.0462	0.0382	0.0181	0.0088	0.0074	0.0031
23	0.7954	0.6342	0.5067	0.4057	0.3256	0.2618	0.2109	0.1703	0.1378	0.1117	0.0907	0.0738	0.0609	0.0491	0.0402	0.0329	0.0151	0.0071	0.0059	0.0024
24	0.7876	0.6217	0.4919	0.3901	0.3101	0.2470	0.1971	0.1577	0.1264	0.1015	0.0817	0.0659	0.0532	0.0431	0.0349	0.0284	0.0126	0.0057	0.0047	0.0018
25	0.7798	0.6095	0.4776	0.3751	0.2953	0.2330	0.1842	0.1460	0.1160	0.0923	0.0736	0.0588	0.0471	0.0378	0.0304	0.0245	0.0105	0.0046	0.0038	0.0014
30	0.7419	0.5521	0.4120	0.3083	0.2314	0.1741	0.1314	0.0994	0.0754	0.0573	0.0437	0.0334	0.0256	0.0196	0.0151	0.0116	0.0042	0.0016	0.0012	*
35	0.7059	0.5000	0.3554	0.2534	0.1813	0.1301	0.0937	0.0676	0.0490	0.0356	0.0259	0.0189	0.0139	0.0102	0.0075	0.0055	0.0017	0.0005	*	*
36	0.6989	0.4902	0.3450	0.2437	0.1727	0.1227	0.0875	0.0626	0.0449	0.0323	0.0231	0.0169	0.0123	0.0089	0.0065	0.0048	0.0014	*	*	*
40	0.6717	0.4529	0.3066	0.2083	0.1420	0.0972	0.0668	0.0460	0.0318	0.0221	0.0154	0.0107	0.0075	0.0053	0.0037	0.0026	0.0007	*	*	*
50	0.6080	0.3715	0.2281	0.1407	0.0872	0.0543	0.0339	0.0213	0.0134	0.0085	0.0054	0.0035	0.0022	0.0014	0.0009	0.0006	*	*	*	*

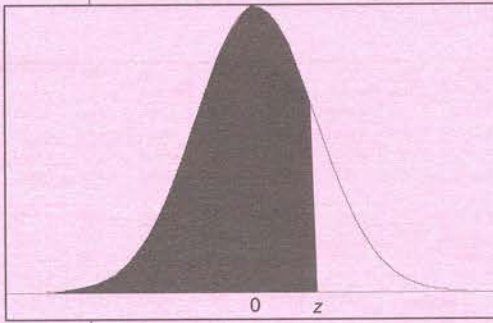
Present Value Interest factors for Annuity of 1 Discounted at  $r$  Percent for  $n$  Periods:

$$PVIFA_{r,n} = [1 - 1 / (1+r)^n] / r$$

Period	1%	2%	3%	4%	5%	6%	7%	8%	9%	10%	11%	12%	13%	14%	15%	16%	20%	24%	25%	30%
1	0.9901	0.9804	0.9709	0.9615	0.9524	0.9434	0.9346	0.9259	0.9174	0.9091	0.9009	0.8929	0.8850	0.8772	0.8696	0.8621	0.8333	0.8065	0.8000	0.7692
2	1.9704	1.9416	1.9135	1.8861	1.8594	1.8334	1.8080	1.7833	1.7591	1.7355	1.7125	1.6901	1.6681	1.6467	1.6257	1.6052	1.5278	1.4508	1.4400	1.3609
3	2.9410	2.8839	2.8286	2.7751	2.7232	2.6736	2.6243	2.5771	2.5313	2.4869	2.4437	2.4018	2.3612	2.3216	2.2832	2.2459	2.1665	1.9813	1.9520	1.8161
4	3.9020	3.8077	3.7171	3.6299	3.5460	3.4651	3.3872	3.3121	3.2397	3.1699	3.1024	3.0373	2.9745	2.9137	2.8550	2.7982	2.5887	2.4043	2.3616	2.1662
5	4.8534	4.7135	4.5797	4.4518	4.3295	4.2124	4.1002	3.9927	3.8897	3.7908	3.6959	3.6048	3.5172	3.4331	3.3522	3.2743	2.9906	2.7454	2.6893	2.4356
6	5.7955	5.6014	5.4172	5.2421	5.0757	4.9173	4.7665	4.6229	4.4859	4.3553	4.2305	4.1114	3.9975	3.8887	3.7845	3.6847	3.3255	3.0205	2.9514	2.6427
7	6.7282	6.4720	6.2303	6.0021	5.7864	5.5824	5.3893	5.2064	5.0330	4.8684	4.7122	4.5638	4.4223	4.2863	4.1548	4.0276	3.6046	3.2423	3.1611	2.8021
8	7.6517	7.3255	7.0197	6.7327	6.4632	6.2098	5.9713	5.7466	5.5348	5.3349	5.1461	4.9676	4.7988	4.6389	4.4873	4.3436	3.8372	3.4212	3.3289	2.9247
9	8.5660	8.1622	7.7861	7.4353	7.1078	6.8017	6.5152	6.2469	5.9952	5.7590	5.5370	5.3282	5.1317	4.9464	4.7716	4.6065	4.0310	3.5855	3.4631	3.0190
10	9.4713	8.9826	8.5302	8.1019	7.7217	7.3601	7.0236	6.7101	6.4177	6.1446	5.8892	5.6502	5.4262	5.2161	5.0188	4.8332	4.1925	3.6819	3.5705	3.0915
11	10.368	9.7868	9.2548	8.7605	8.3064	7.8869	7.4987	7.1390	6.8052	6.4951	6.2065	5.9377	5.6869	5.4527	5.2337	5.0286	4.3271	3.7757	3.6564	3.1473
12	11.255	10.575	9.9540	9.3851	8.8633	8.3838	7.9427	7.5361	7.1607	6.8137	6.4924	6.1944	5.9176	5.6603	5.4206	5.1971	4.4392	3.8514	3.7251	3.1903
13	12.134	11.348	10.635	9.9856	9.3936	8.8527	8.3577	7.9038	7.4869	7.1034	6.7499	6.4235	6.1218	5.8424	5.5831	5.3423	4.5327	3.9124	3.7801	3.2233
14	13.004	12.106	11.296	10.563	9.8986	9.2950	8.7455	8.2442	7.7862	7.3667	6.9819	6.6282	6.3025	6.0021	5.7245	5.4675	4.6106	3.9616	3.8241	3.2487
15	13.865	12.849	11.938	11.118	10.380	9.7122	9.1079	8.5595	8.0607	7.6061	7.1909	6.8109	6.4624	6.1422	5.8474	5.5755	4.6755	4.0013	3.8593	3.2682
16	14.718	13.578	12.561	11.652	10.838	10.106	9.4466	8.8514	8.3126	7.8237	7.3792	6.9740	6.6039	6.2651	5.9542	5.6685	4.7296	4.0333	3.8874	3.2832
17	15.562	14.292	13.166	12.166	11.274	10.477	9.7632	9.1216	8.5436	8.0216	7.5488	7.1196	6.7291	6.3729	6.0472	5.7487	4.7746	4.0591	3.9099	3.2948
18	16.398	14.992	13.754	12.659	11.690	10.828	10.059	9.3719	8.7556	8.2014	7.7016	7.2497	6.8399	6.4674	6.1280	5.8178	4.8122	4.0799	3.9279	3.3037
19	17.226	15.678	14.324	13.134	12.085	11.158	10.336	9.6036	8.9501	8.3649	7.8393	7.3658	6.9380	6.5504	6.1982	5.8775	4.8435	4.0967	3.9424	3.3105
20	18.046	16.351	14.877	13.590	12.462	11.470	10.594	9.8181	9.1285	8.5136	7.9633	7.4694	7.0248	6.6231	6.2593	5.9288	4.8696	4.1103	3.9539	3.3158
21	18.857	17.011	15.415	14.029	12.821	11.764	10.836	10.017	9.2922	8.6487	8.0751	7.5620	7.1016	6.6870	6.3125	5.9731	4.8913	4.		



## Cumulative Probabilities for the Standard Normal (Z) Distribution



Values in the table correspond to the area under the curve of a standard normal random variable for a value at or below  $z$ .

$z$	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990
3.1	0.9990	0.9991	0.9991	0.9991	0.9992	0.9992	0.9992	0.9992	0.9993	0.9993
3.2	0.9993	0.9993	0.9994	0.9994	0.9994	0.9994	0.9994	0.9995	0.9995	0.9995
3.3	0.9995	0.9995	0.9995	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996	0.9997
3.4	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9998
3.5	0.9998									
4.0	0.99997									
4.5	0.999997									
5.0	0.9999997									