

#### **CIFA PART III SECTION 6**

#### DERIVATIVES ANALYSIS

FRIDAY: 30 November 2018.

Time Allowed: 3 hours.

Answer ALL questions. Marks allocated to each question are shown at the end of the question. Show ALL your workings.

## **QUESTION ONE**

- (a) In relation to derivatives trading, distinguish between "forward commitments" and "contingent claims". (2 marks)
- (b) (i) Explain the term "cross-hedging".

(2 marks)

(ii) Argue three cases against hedging.

(3 marks)

(c) Using illustrative examples, examine two ways of writing a call option.

(4 marks)

(d) Jeremy Cheposin is an equity analyst at ABC Ltd. He believes that call options are an alternative approach to establish a long position on Triple M stock. The current market price of a six month put option with a strike price of Sh.100 is Sh.5.35.

The risk-free interest rates are provided below:

| Maturity | Risk-free interes |
|----------|-------------------|
| 3 months | 0.50              |
| 6 months | <b>05</b> 50      |
| l year   | 1.00              |
| •        |                   |

## Required:

The price of a six-month call option using the put-call parity.

(3 marks)

(e) A stock currently trades at a price of Sh.65 and has an exercise price of Sh.60. The stock price can go up by 20% or down by 17% each period. The risk-free rate is 5%.

# Required:

The price of a call option expiring in two periods using a two-period binomial model.

(6 marks)

(Total: 20 marks)

# **QUESTION TWO**

(a) Three months ago, Zuhura Ltd. purchased a European receiver swaption that is exercisable into a two-year swap with semi-annual payments. The swaption has a semi-annual exercise rate of 2.75% and a notional principal of Sh.25,000,000. The swaption has just expired.

The relevant term structure of interest rate is presented below:

| Days | London Interbank Offered Rate (LIBOR) |
|------|---------------------------------------|
|      | (%)                                   |
| 180  | 1.95                                  |
| 360  | 3.68                                  |
| 540  | 4.11                                  |
| 720  | 4.65                                  |

# Required:

The market value of the receiver swaption.

(4 marks)

(b) SM Bank entered into a Sh.5 million, 1 year equity swap with quarterly payments 300 days ago. The bank agreed to pay an annual fixed rate of 4% and receive the return on an international equity index. The index was trading at 3000 at the end of the third quarter, 30 days ago. The current 60-day London Interbank Offered Rate (LIBOR) rate is 3.6%. The discount factor is 0.9940 and the index is now at 3150.

### Required:

The value of the swap to the bank.

(4 marks)

(c) After examining its long-term liabilities, Lake Bank Limited has decided that it needs to borrow Sh.100 million over the next two years to finance its operations. For this type of funding, Lake Bank Limited issues quarterly coupon short-term Floating Rate Notes (FRN) based on 90-day London Interbank Offered Rate (LIBOR). The bank is concerned that interest rates may shift upwards and is considering using interest rate derivatives. The managers at the bank have collected quotes on the over-the-counter (OTC) interest rate caps and floors from the markets based on a notional principal of Sh.100 million.

#### Interest rate caps and floors

|              |         |             | Interest r      | ate caps    | Interest rate floors |             |  |
|--------------|---------|-------------|-----------------|-------------|----------------------|-------------|--|
| Term (Years) | LIBOR   | Settlement  | <b>Rate (%)</b> | Price (Sh.) | Rate (%)             | Price (Sh.) |  |
| 1            | 90-day  | Quarterly   | 3.50            | 2,000,000   | 2.55                 | 1,900,000   |  |
| 1            | 180-day | Semi-annual | 3.50            | 2,000,000   | 2.55                 | 1,900,000   |  |
| 2            | 90-day  | Quarterly   | 3.65            | 2,200,000   | 2.70                 | 2,090,000   |  |
| 2            | 180-day | Semi-annual | 3.65            | 2,200,000   | 2.70                 | 2,090,000   |  |

## Required:

(i) Explain the term "interest rate collar".

(1 mark)

- (ii) The payoff from this derivative 360-days after the contract initiation assuming that the LIBOR at expiration is expected to be 3.75%.
- (iii) The expected payoff after 720-days from a short position is the 2-year semi-annual interest rate floor assuming that the LIBOR at expiration is expected to be 240% (3 marks)
- (d) On 1 March 2018, the one-month London Interbank Offered Rate (LIBOR) was 5.50% and the two-month LIBOR rate was 6.00%. The April treasury futures were quoted at 93.75. The contract size was Sh.5,000,000. The one-month LIBOR rate observed on 1 April 2018 was 7.25%.

(Assume that there is no basis risk and that one year has 360 days).

#### Required

Determine whether an arbitrage opportunity exists.

(5 marks)

(Total: 20 marks)

# **QUESTION THREE**

(a) Assess four objectives of global regulation of derivatives market.

(8 marks)

(b) John Njoroge is a derivative consultant in New York and is working on four assignments relating to different clients.

## Client 1:

The client manages equity portfolio for a pension fund. One month (30 days) ago, the pension fund expected a large inflow of cash in 60 days. In order to hedge against a potential rise in equity value, Njoroge advised the client to enter into a long forward contract on the S & P 500 index expiring in 60 days. The information relating to the transaction is provided below:

| Price of a 60-day S & P 500 forward contract 30 days ago        | 1,403.22 |
|---|----------|
| S & P 500 index level today                                     | 1,450.82 |
| Annualised continuously compounded risk-free rate               | 3.92%    |
| Annualised continuously compounded dividend yield for S & P 500 | 2.50%    |

# Client 2:

Three months ago (90 days), the client purchased a bond with a 5% annual coupon rate and a maturity of 7 years from the date of purchase. The bond has a face value of Sh.1,000 and pays interest every 180 days from the date of issue. As the client is concerned about the potential increase in interest rate, Njoroge advised the client to enter into a short forward contract expiring in 360 days. The annualised risk-free rate now is 4% per year and the price of the bond with accrued interest is Sh.1,071.33.

## Client 3:

A corporate treasurer has gathered the following information:

| Annualised 90 day LIBOR rate                   | 3.2% |
|--|------|
| Annualised 450 day LIBOR rate                  | 4.5% |
| Annualised risk-free rate in the United States | 4.0% |
| Annualised risk-free rate in the Euro zone     | 6.0% |
| Spot exchange rate, USD per EUR                | 1.39 |

Three months (90 days) from now, the treasurer expects to borrow USD 5 million at LIBOR for a period of twelve months (360 days). He is concerned that interest rates may rise significantly over the next few months and wishes to hedge this risk. Njoroge advises him to enter into a forward rate agreement (FRA) expiring in 90 days on a 360 day LIBOR.

#### Client 4:

The client expects an inflow of EUR 3,000,000 that needs to be converted to United States Dollars (USD) in 270 days and is concerned that the Euro will decline in value over this period. Njoroge advises the client to enter into an agreement to sell the Euro forward in 270 days.

# Required:

(i) The value of the equity forward contract.

(3 marks)

(ii) The price of the forward contract on the bond purchased.

(3 marks)

(iii) The rate on the forward rate agreement (FRA) expiring in 90 days on 360 day LIBOR.

(3 marks)

(iv) The forward price that the client should sell the Euros.

(3 marks)

(Total: 20 marks)

# **QUESTION FOUR**

(a) Highlight three similarities between "forward contracts" and "futures contracts".

(3 marks)

(b) Explain the term "marking to market" as used in futures market.

(2 marks)

(c) Agva Asset Management Group (AAMG) is a pension fund management firm. One of its funds consists of Sh.300 million allocated 80% to equities and 20% to bonds. The equity portion has a beta of 1.10 and the bond portion has a duration of 6.5. AAMG would like to temporarily adjust the asset allocation to 50% equities and 50% bonds. The firm will use stock index futures and bond futures to achieve this objective. The stock index futures contract has a price of Sh.200,000 and a beta of 0.96. The bond futures contract has an implied modified duration of 7.2 and a price of Sh.105,250. The yield beta is 1.0. The transaction will be put in place on 15 November 2018, and the horizon date for termination is 10 January 2019.

### Required:

- (i) The number of stock index futures contracts that AAMG should sell to achieve the set objective.

  (3 marks)
- (ii) The number of box futures contracts that AAMG should buy to achieve the set objective. (3 marks)
- (d) Martin Opondo believes that the stock price of XYZ Ltd. will have little volatility over the next three months. He wants to construct a butterfly spread option strategy to take advantage of the opportunity he believes exists.

The following data show 3-month options which are available on XYZ Ltd.'s stock:

| Option | Strike price (Sh.) | Option price (Sh.) |
|--------|--------------------|--------------------|
| Put    | 35                 | 1.25               |
| Put    | 40                 | 3.50               |
| Put    | 45                 | 5.50               |
| Call   | 40                 | 5.90               |

Martin can use any number of contracts of the above options to construct his strategy.

#### Required:

The total profit (loss) on a properly constructed butterfly spread, assuming that the price of the underlying stock at expiration is Sh.41. (4 marks)

Evans Nyongesa has recently opened a margin account in which he trades wheat futures. In July 2018, Nyongesa entered a long position of five wheat contracts, each of which covered 5,000 bushel.

The contract price was Sh.2 and each contract required an initial margin deposit of Sh.150 and maintenance of Sh.100.

On day 1, the price of wheat declined by Sh.0.02.

On day 2, the price of wheat increased by Sh.0.01.

On day 3, the price of wheat declined by Sh.0.03.

Required:

Determine the margin balance for this position at the end of day 3.

(5 marks)

(Total: 20 marks)

#### **QUESTION FIVE**

(a) James Kivuva sells a September 2018 call on Delta Ltd.'s shares with an exercise price of Sh.45 for a Sh.3 premium. He also buys a September 2018 call on the same Delta Ltd.'s shares with an exercise price of Sh.40 for a Sh.5 premium.

## Required:

(i) Identify with a reason, the type of option strategy employed by James Kivuva.

(1 mark)

(ii) Determine the maximum profit and loss for James Kivuva.

(2 marks)

(b) An investor wishes to purchase a European put option which has the following characteristics:

| Current market price of a share                   | Sh. 25 |
|---|--------|
| Strike price for a six months put option          | Sh. 20 |
| Annual standard deviation of the underlying stock | 25%    |
| Current continuously compounded risk-free rate    | 4.25%  |
| $N(d_1)$  | 0.9737 |
| $N(d_2)$  | 0.9651 |
|   |        |

#### Required:

The value of the put option using the Black-Scholes-Merton (BSM) model.

(3 marks)

Josovina Investment Bank has Sh.400 million portfolio available for investment. The cost of funds is 5.5%. The bank lends 50% of the assets to domestic customers for an average annual interest rate of 7.35%. The balance of the portfolio is lent to some Ugandan clients at an annual interest rate of 8%. The spot exchange rate is KES 0.0266/UGX.

At the same time, the bank sells a forward contract to eliminate exchange rate risk equal to the expected receipts one year from now.

The forward exchange rate is KES 0.0250/UGX.

## Required:

The net interest margin on the balance sheet of Josovina Investment Bank.

(4 marks)

(d) (i) Explain the term waption" in the context of derivatives.

(1 mark)

(ii) Examine three primary uses of swaptions.

(3 marks)

(e) Jedi Limited, a Japanese company issues a bond with a face value of 1.2 billion Japanese Yen (¥) and a coupon rate of 5.25%. The company is contemplating to use a swap to convert this bond into a Euro (€) denominated bond.

# Additional information:

- 1. The current exchange rate is ¥ 120/€.
- 2. The fixed rate on Euro-denominated swaps is 6%.
- 3. The fixed rate on Yen-denominated swaps is 5%.
- 4. All payments will be made annually, so there is no adjustment such as Days/360.

## Required:

(i) Describe the terms of the swap between the two counter parties.

(1 mark)

(ii) Determine the cash flow at the start of the contract.

(1 mark)

(iii) Calculate all interest cash flows at each interest payment date.

(2 marks)

(iv) Calculate all principal cash flows at the maturity of the bond.

(2 marks) (Total: 20 marks)

Present Value of 1 Received at the End of n Periods:

| PVIF, | == | 1/(1 | +r)" | =(1 | +r)·" |
|-------|----|------|------|-----|-------|
|       | 41 |      | ,    | `   |       |

| Period | 1%    | 2%    | 3%    | 4%    | 5%    | 6%    | 7%    | 8%    | 9%    | 10%   | 12%   | 14%   | 15%   | 16%   | 18%    | 20%   | 24%   | 28%   | 32%   | 36%   |
|--------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|--------|-------|-------|-------|-------|-------|
| 1      | .9901 | .9804 | .9709 | .9615 | .9524 | .9434 | .9346 | .9259 | .9174 | .9091 | .8929 | 8772  | .8696 | .8621 | .8475  | .8333 | .8065 | .7813 | .7576 | .7353 |
| 2      | .9803 | .9612 | .9426 | .9246 | .9070 | .8900 | 8734  | .8573 | .8417 | .8264 | .7972 | .7695 | 7561  | .7432 | .7182  | .6944 | .6504 | .6104 | 5739  | .5407 |
| 3      | .9706 | .9423 | .9151 | .8890 | .8638 | .8396 | .0163 | .7938 | .7722 | .7513 | .7118 | .6750 | .6575 | .6407 | .6086  | .5787 | .5245 | .4768 | 4348  | 3975  |
| 4      | .9610 | .9238 | .8885 | .8548 | .8227 | .7921 | .7629 | .7350 | .7084 | .6830 | .6355 | .5921 | .5718 | .5523 | .5158  | .4823 | .4230 | .3725 | .3294 | .2923 |
| 5      | .9515 | .9057 | .8626 | .8219 | .7835 | .7473 | .7130 | .6806 | .6499 | .6209 | .5674 | 5194  | .4972 | .4761 | .4371  | .4019 | .3411 | .2910 | .2495 | .2149 |
| 6      | 9420  | .8880 | .8375 | .7903 | .7462 | .7050 | :6663 | .6302 | .5963 | .5645 | .5066 | .4556 | .4323 | .4104 | .3704  | .3349 | .2751 | .2274 | .1890 | .1580 |
| 7      | .9327 | .8706 | .8131 | .7599 | .7107 | .6651 | .6227 | .5835 | .5470 | .5132 | .4523 | .3996 | .3759 | .3538 | .3139  | .2791 | .2218 | :1776 | .1432 | .1162 |
| 8      | .9235 | .8535 | .7894 | .7307 | .6768 | .6274 | .5820 | .5403 | .5019 | .4665 | .4039 | .3506 | .3269 | .3050 | .2660  | .2326 | .1789 | .1388 | .1085 | .0854 |
| 9      | .9143 | .8368 | .7664 | .7026 | .6446 | .5919 | .5439 | .5002 | .4604 | .4241 | .3606 | 3075  | .2843 | .2630 | .2255  | .1938 | .1443 | .1084 | .0822 | .0628 |
| 10     | .9053 | .8203 | .7441 | .6756 | .6139 | .5584 | .5083 | .4632 | .4224 | .3855 | .3220 | .2697 | .2472 | .2267 | .1911  | .1615 | .1164 | .0847 | .0623 | .0462 |
| . 11   | .8963 | .8043 | .7224 | .6496 | .5847 | .5268 | .4751 | .4289 | .3875 | .3505 | .2875 | .2366 | .2149 | .1954 | .1619  | .1346 | .0938 | .0662 | .0472 | .0340 |
| 12     | ,8874 | .7885 | .7014 | .6246 | .5568 | .4970 | .4440 | .3971 | .3555 | .3186 | .2567 | .2076 | .1869 | 1685  | .1372  | .1122 | .0757 | .0517 | .0357 | .0250 |
| 13     | .8787 | .7730 | .6810 | .6006 | .5303 | .4688 | .4150 | .3677 | .3262 | .2897 | .2292 | .1821 | .1625 | .1452 | .1163  | .0935 | .0610 | .0404 | .0271 | .0184 |
| 14     | .8700 | .7579 | .6611 | .5775 | .5051 | .4423 | .3878 | .3405 | .2992 | .2633 | .2046 | .1597 | .1413 | .1252 | .0985  | .0779 | .0492 | .0316 | .0205 | .0135 |
| 15     | .8613 | .7430 | .6419 | .5553 | .4810 | .4173 | .3624 | 3152  | .2745 | .2394 | .1827 | .1401 | .1229 | .1079 | .0835  | .0649 | .0397 | .0247 | .0155 | 0099  |
| 16     | .8528 | .7284 | .6232 | .5339 | .4581 | .3936 | .3387 | .2919 | .2519 | .2176 | .1631 | .1229 | 1069  | .0930 | .0708  | .0541 | .0320 | .0193 | .0118 | .0073 |
| 17     | .8444 | .7142 | .6050 | .5134 | .4363 | .3714 | .3166 | .2703 | .2311 | .1978 | .1456 | .1078 | .0929 | .0802 | .0600  | .0451 | .0258 | .0150 | .0089 | .0054 |
| 18     | .8360 | .7002 | .5674 | .4936 | .4155 | .3503 | .2959 | .2502 | .2120 | .1799 | .1300 | .0946 | .0808 | .0691 | .0508  | .0376 | .0208 | .0118 | .0068 | .0039 |
| 19     | .8277 | .6864 | .5703 | .4746 | .3957 | .3305 | .2765 | .2317 | .1945 | .1635 | .1161 | .0829 | .0703 | .0596 | .0031  | .0313 | .0168 | .0092 | .0051 | .0029 |
| 20     | .8195 | .6730 | .5537 | .4564 | 3769  | .3118 | .2584 | .2145 | .1784 | .1486 | 1037  | .0728 | .0611 | .0514 | 2.0365 | .0261 | .0135 | .0072 | .0039 | .0021 |
| 25     | 7798  | .6095 | .4776 | .3751 | .2953 | .2330 | .1842 | .1460 | .1160 | .0923 | .0588 | .0378 | .0304 | .0245 | .0160  | .0105 | .0046 | .0021 | .0010 | 0005  |
| 30     | 7419  | .5521 | .4120 | .3083 | .2314 | .1741 | .1314 | .0994 | .0754 | .0573 | .0334 | .0196 | .0151 | 0116  | .0070  | .0042 | .0016 | .0006 | .0002 | .000  |
| 40     | .6717 | 4529  | 3066  | .2083 | 1420  | .0972 | .0668 | .0460 | .0318 | .0221 | .0107 | .0053 | .0037 | .0026 | .0013  | .0007 | .0002 | .0001 |       |       |
| 50     | .6080 | .3715 | .2281 | .1407 | .0872 | .0543 | .0339 | .0213 | .0134 | .0085 | .0035 | .0014 | 0009  | .0006 | .0003  | .0001 |       |       |       |       |
| 60     | .5504 | .3048 | .1697 | .0951 | .0535 | .0303 | .0173 | .0099 | .0057 | .0033 | .0011 | .0004 | 50002 | .0001 |        | •     | •     | •     | -     |       |

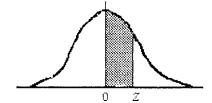
Present Value of an Annuity of 1 Per Period for n Periods:  

$$PVIF_{rt} = \sum_{t=1}^{n} \frac{1}{(1+r)^t} = \frac{1}{(1+r)^t}$$

| NUMBER 31 |         |         |         |          |          |         | - 0      |          |          |           |        |        |        |        |        |                  |           |        |                  |
|-----------|---------|---------|---------|----------|----------|---------|----------|----------|----------|-----------|--------|--------|--------|--------|--------|------------------|-----------|--------|------------------|
| payments  | 1%      | 2%      | 3%      | 4%       | 5%       | 6%      | 5°7%     | 8%       | 9%       | 10%_      | 12%    | 14%    | 15%    | 16%    | 18%    | 20%              | 24%       | 28%    | 32%              |
| 1         | 0.9901  | 0.9804  | 0.9709  | 0.9615   | 0.9524   | 0.9434  | 0.9346   | 0.9259   | 0.9174   | 0.9091    | 0.8929 | 0.8772 | 0.8696 | 0.8621 | 0.8475 | 0.8333           | 0.8065    | 0.7813 | 0.7576           |
| 2         | 1.9704  | 1.9416  | 1.9135  | 1.8861   | 1.8594   | 1,8334  | 1.8080   | 1.7833   | 1.7591   | 1.7355    | 1.6901 | 1.6467 | 1.6257 | 1.6052 | 1.5656 | 1.5278           | 1,4568    | 1.3916 | 1.3315           |
| 3         | 2.9410  | 2.8839  | 2,8286  | 2.7751   | 2.7232   | 2.6730  | 2.6243   | 2.5771   | 2.5313   | 2.4869    | 2,4018 | 2.3216 | 2.2832 | 2.2459 | 2.1743 | 2,1065           | 1.9813    | 1.8684 | 1.7663           |
| 4         | 3.9020  | 3.8077  | 3.7171  | 3.6299   | 3.5460   | 3.4651  | 3.3872   | 3.3121   | 3.2397   | 3.1699    | 3.0373 | 2.9137 | 2.8550 | 2.7982 | 2.6901 | 2.5887           | 2.4043    | 2.2410 | 2.0957           |
| 5         | 4.8534  | 4.7135  | 4.5797  | 4.4518   | 4/23295  | 4.2124  | 4.1002   | 3.9927   | 3.8897   | 3.7908    | 3.6048 | 3.4331 | 3.3522 | 3.2743 | 3.1272 | 2.9906           | 2.7454    | 2.5320 | 2.3452           |
|           |         |         |         | _(       | 25       |         |          |          |          |           |        |        |        |        |        |                  |           |        |                  |
| 6         | 5.7955  | 5.6014  | 5.4172  | 5.2421   | 5.0757   | 4.9173  | 4.7665   | 4.6229   | 4.4859   | 4.3553    | 4.1114 | 3.8887 | 3.7845 | 3.6847 | 3.4976 | 3.3255           | 3.0205    | 2.7594 | 2.5342           |
| 7         | 6.7282  | 6.4720  | 6,2303  | 6,0021   | 5.7864   | 5.5824  | 5.3893   | 5.2064   | 5.0330   | 4.8684    | 4.5638 | 4.2883 | 4.1604 | 4.0386 | 3.8115 | 3.6046           | 3.2423    | 2.9370 | 2.6775           |
| 8         | 7.6517  | 7.3255  | 7.0197  | , 6.7327 | 6.4632   | 6.2098  | 5.9713   | 5.7466   | 5.5348   | 5.3349    | 4.9676 | 4.6389 | 4.4873 | 4.3436 | 4.0776 | 3.8372           | 3.4212    | 3.0758 | 2.7860           |
| 9         | 8.5660  | 8.1622  | 7.7861  | 7.4353   | 7.1078   | 6.8017  | 6.5152   | 6.2469   | 5.9952   | 5.7590    | 5.3282 | 4.9464 | 4.7716 | 4.6065 | 4.3030 | 4.0310           | 3.5655    | 3.1842 | 2.8681           |
| 10        | 9.4713  | 8,9826  | 8.5302  | 8.1109   | 7.7217   | 7.3601  | 7.0236   | 6.7101   | 6.4177   | 6.1446    | 5,6502 | 5.2161 | 5.0188 | 4.8332 | 4.4941 | 4.1925           | 3.6819    | 3.2689 | 2.9304           |
| 11        | 10 3676 | 9.7868  | 9.2526  | 8.7605   | 8.3064   | 7.8869  | 7.4987   | 7.1390   | 6.8052   | 6,4951    | 5.9377 | 5.4527 | 5.2337 | 5.0286 | 4.6560 | 4.3271           | 3.7757    | 3.3351 | 2.9776           |
| 12        | 11.2551 | 10.5753 |         | 9.3851   | 8.8633   | 8.3838  | 7.9427   | 7.5361   | 7.1607   | 6.8137    | 6.1944 | 5.6603 | 5.4206 | 5.1971 | 4.7932 | 4.4392           | 3.8514    | 3.3868 | 3.0133           |
| 13        |         | 11.3484 |         |          | 9.3936   | 8.8527  | 8.3577   | 7.9038   | 7.4869   | 7.1034    | 6.4235 | 5.8424 | 5.5831 | 5.3423 | 4.9095 | 4.5327           | 3.9124    | 3.4272 | 3.0404           |
| 14        |         | 12.1062 |         |          |          | 9.2950  | 8.7455   | 8.2442   | 7.7862   | 7.3667    | 6.6282 | 6.0021 | 5.7245 | 5.4675 | 5.0081 | 4.6106           | 3.9616    | 3.4272 |                  |
| 15        |         | 12.8493 |         |          |          |         |          | 8.5595   | 8.0607   |           | 6.8109 | 6.1422 | 5.8474 | 5.5755 |        | 4.6755           | 4.0013    | 3.4834 | 3.0609<br>3.0764 |
|           |         |         |         |          |          | ******  | 0.7010   | 0.0000   | 0.0001   | 7,0001    | 0.0103 | 9.1722 | 3.0414 | 3,3133 | 3.0310 | 4.6133           | 4.0013    | 3.4034 | 3.0764           |
| 16        | 14.7179 | 13.5777 | 12.5611 | 11.6523  | 10.8378  | 10.1059 | 9,4466   | 8.8514   | 8.3126   | 7.8237    | 6.9740 | 6.2651 | 5.9542 | 5.6685 | 5.1624 | 4.7296           | 4.0333    | 3.5026 | 3.0882           |
| 17        | 15.5623 | 14.2919 | 13,1661 | 12.1657  | 11.2741  | 10.4773 | 9.7632   | 9.1216   | 8.5436   | 8.0216    | 7.1196 | 6.3729 | 6.0472 | 5.7487 | 5.2223 | 4.7746           | 4.0591    | 3.5177 | 3.0971           |
| 18        | 16.3983 | 14.9920 | 13.7535 | 12.6593  | 11.6896  | 10.8276 | 10.0591  | 9.3719   | 8.7556   | 8.2014    | 7.2497 | 6.4674 | 6.1280 | 5,8178 | 5.2732 | 4.8122           | 4.0799    | 3.5294 | 3 1039           |
| 19        | 17.2260 | 15.6785 | 14.3238 | 13.1339  | 12.0853  | 11.1581 | 10.3356  | 9,6036   | 8.9501   | 8.3649    | 7.3658 | 6.5504 | 6,1982 | 5.8775 | 5.3162 | 4.8435           | 4.0967    | 3.5386 | 3.1090           |
| 20        | 18.0456 | 16.3514 | 14.8775 | 13.5903  | 12.4622  | 11.4699 | 10.5940  | 9.8181   | 9.1285   | 8.5136    | 7.4694 | 6.6231 | 6.2593 | 5.9288 | 5.3527 | 4.8696           | 4.1103    | 3.5458 | 3 1129           |
| 25        | 22 0232 | 19.5235 | 17 4131 | 15 6221  | 14 0939  | 12 7834 | 11 6536  | 10 6748  | 9 8226   | 9.0770    | 7.8431 | 6.8729 | 6.4641 | 6.0971 | 5.4660 | 4,9476           | 4 4 4 7 4 | 3.5540 | 3 4 9 9 9        |
| . 30      |         | 22.3965 |         |          |          |         |          |          |          |           | 8.0552 | 7.0027 | 6.5660 | 6.1772 | 5.5168 | 4.9789           |           | 3.5640 | 3.1220           |
| 40        |         | 27.3555 |         |          |          |         |          |          |          |           | 8,2438 | 7.1050 | 6.6418 | 6,2335 | 5.5482 |                  | 4.1601    | 3.5693 | 3 1242           |
| 50        |         | 31.4236 |         |          |          |         |          |          |          |           | 8.3045 | 7.1030 | 6.6605 | 6.2333 | 3.5541 | 4.9966           | 4.1659    | 3.5712 | 3.1250           |
| 60        |         | 34.7609 |         |          |          |         |          |          |          |           |        | 7.1327 | 6.6651 | 6.2402 | 5 5553 | 4.9995<br>4.9999 | 4.1666    | 3.5714 | 3.1250           |
|           |         | 54,7003 | 27,0100 |          | . 0.5230 | .0.1014 | . 7.0032 | . 2.5736 | 11.07400 | J. 30 1 Z | 4.5240 | 7.1401 | 9.0031 | 0.2402 | J 2323 | +.9999           | 4.1667    | 3.5714 | 3 1250           |

# NORMAL CURVE

AREAS
under the
STANDARD
NORMAL CURVE
from 0 to z



| z   | 0     | 1              | 2              | 3     | 4      | 5              | 6              | 7 .            | 8     | 9     |
|-----|-------|----------------|----------------|-------|--------|----------------|----------------|----------------|-------|-------|
| 0.0 | .0000 | .0040          | .0080          | .0120 | .0160  | .0199          | .0239          | .0279          | .0319 | .0359 |
| 0.1 | .0398 | .0438          | .0478          | .0517 | .0557  | .0596          | .0636          | .0675          | .0714 | .0754 |
| 0.2 | .0793 | .0832          | .0871          | .0910 | .0948  | .0987          | .1026          | .1064          | .1103 | .1141 |
| 0.3 | .1179 | .1217          | .1255          | .1293 | .1331  | .1368          | .1406          | .1443          | .1480 | .1517 |
| 0.4 | .1554 | .1591          | .1628          | .1664 | .1700  | .1736          | .1772          | .1808          | .1844 | .1879 |
|     |       |                |                |       |        |                |                |                |       |       |
| 0.5 | .1915 | .1950          | .1985          | .201  | .2051  | .2088          | .2123          | .2157          | .2190 | .2224 |
| 0.6 | .2258 | .2291          | .2324          | .2357 | .2389  | .2422          | .2454          | .2486          | .2518 | .2549 |
| 0.7 | .2580 | .2612          | .2642          | .2673 | .2704  | .2734          | .2704          | .2794          | .2823 | .2852 |
| 8.0 | .2881 | .2910          | .2939          | .2967 | .2996  | .3023          | .3051          | .3078          | .3106 | 3133  |
| 0.9 | .3159 | .3186          | .3212          | .3238 | .3264  | .3289          | .3315          | .3340          | .3365 | .3389 |
|     |       |                |                |       |        |                |                | OE/3           |       |       |
| 1.0 | .3413 | .3438          | .3461          | .3485 | .3508  | .3531          | .3554          | 3577           | .3599 | .3621 |
| 1.1 | .3643 | .3665          | .3686          | .3708 | .3729  | .3749          | .37705         | .3790          | .3810 | .3830 |
| 1.2 | .3849 | .3869          | .3888          | .3907 | .3925  | .3944          | .3962          | .3980          | .3997 | .4015 |
| 1.3 | .4032 | .4049          | .4066          | .4082 | .4099  | .4115          | <b>€</b> 7131  | .4147          | .4162 | .4177 |
| 1.4 | .4192 | .4207          | .4222          | .4236 | .4251  | .4265          | .4279          | .4292          | .4306 | .4319 |
|     |       |                |                |       |        | مابات          |                |                |       |       |
| 1.5 | .4332 | .4345          | .4357          | .4370 | .4382  | 4394           | .4406          | .4418          | .4429 | .4441 |
| 1.6 | .4452 | .4463          | .4474          | .4484 | .4495  | 4.4505         | .4515          | .4525          | .4535 | .4545 |
| 1.7 | .4554 | .4564          | .4573          | .4582 | .45910 |                | .4608          | .4616          | .4625 | .4633 |
| 1.8 | .4641 | .4649          | .4656          | .4664 | .4671  | .4678          | .4686          | .4693          | .4699 | .4706 |
| 1.9 | .4713 | .4719          | .4726          | .4732 | 4738   | .4744          | .4750          | .4756          | .4761 | .4767 |
| 2.0 | .4772 | .4778          | .4783          | .4788 | .4793  | .4798          | .4803          | .4808          | .4812 | .4817 |
| 2.1 | .4821 | .4826          | .4830          | 4834  | .4838  | .4842          | .4846          | .4850          | 4854  | 4857  |
| 2.2 | .4861 | .4864          | .4868          |       | .4875  | .4878          | .4881          | .4884          | .4887 | .4890 |
| 2.3 | .4893 | .4896          | 4898           | .4901 | .4904  | .4906          | .4909          | .4911          | .4913 | .4916 |
| 2.4 | .4918 | .4920          | 4922           | .4925 | .4927  | .4929          | .4931          | .4932          | .4934 | .4936 |
|     |       | ×              | 000            |       |        |                |                |                |       |       |
| 2.5 | .4938 | .494%          | .4941          | .4943 | .4945  | .4946          | .4948          | .4949          | .4951 | .4952 |
| 2.6 | .4953 | .4955          | .4956          | .4957 | .4959  | .4960          | .4961          | .4962          | .4963 | .4964 |
| 2.7 | .4965 | <b>%</b> 4966  | .4967          | .4968 | .4969  | .4970          | .4971          | .4972          | .4973 | .4974 |
| 2.8 | .4974 | .4975          | .4976          | .4977 | .4977  | .4978          | .4979          | .4979          | .4980 | .4981 |
| 2.9 | .4981 | .4982          | .4982          | .4983 | .4984  | .4984          | .4985          | .4985          | .4986 | .4986 |
|     |       |                |                |       |        |                |                |                |       |       |
| 3.0 | .4987 | .4987          | .4987          | .4988 | .4988  | .4989          | .4989          | .4989          | .4990 | .4990 |
| 3.1 | .4990 | .4991          | .4991          | .4991 | .4992  | .4992          | .4992          | .4992          | .4993 | .4993 |
| 3.2 | .4993 | .4993          | .4994          | .4994 | .4994  | .4994          | .4994          | .4995          | .4995 | .4995 |
| 3.3 | .4995 | .4995          | .4995          | .4996 | .4996  | .4996          | .4996          | .4996          | .4996 | .4997 |
| 3.4 | .4997 | .4997          | .4997          | .4997 | .4997  | .4997          | .4997          | .4997          | .4997 | .4998 |
| 3.5 | .4998 | .4998          | .4998          | .4998 | .4998  | .4998          | .4998          | .4998          | .4998 | .4998 |
|     | .4998 |                | .4999          | .4999 | .4999  | .4999          | .4999          | .4990<br>.4999 | .4999 | .4999 |
| 3.6 | 1     | .4998          | .4999          | .4999 | .4999  |                | .4999          | .4999<br>.4999 | .4999 | .4999 |
| 3.7 | .4999 | .4999<br>.4999 | .4999<br>.4999 | .4999 | .4999  | .4999<br>.4999 | .4999<br>.4999 | .4999<br>.4999 | .4999 | .4999 |
| 3.8 | .4999 | .4999<br>.5000 | .5000          | .5000 | .5000  | .5000          |                | .4999<br>.5000 | .5000 | .5000 |
| 3.9 | .5000 |                | .5000          | .5000 | .5000  | .5000          | .5000          | .5000          | .5000 | .5000 |