THE KENYA NATIONAL EXAMINATIOS COUNCIL
nya Certificat of cond ry Education

121/ **Paper I**

MATHEMATICS ALT A ©

Dec. 2022 ½ ho

am ....

Candidate's Signatu .... Dat ....

In tructions to Candidates

(a) Write your name and index number in the spaces provided above.
(b) Sign and write the date of examination in the spaces provided above.
(c) This paper consists of **two** sections: **Section I** and **Section II.**
(d) Answer **all** the questions in **Section I** and only **five** questions from **Section II.**
(e) **Show all the steps in your calculation, giving your answers at each stage in the spaces provided below each question.**
(f) Marks may be given for correct working even if the answer is wrong.
(g) **Non-programmable** silent electronic calculators and KNEC mathematical tables may be used, except where stated otherwise.
(h) **This paper consists of 19 printed pages.**
(i) **Candidates should check the question paper to ascertain that all the pages are printed as indicated and that no questions are missing.**
(j) **Candidates should answer the questions in English.**

For Examiner's Use Only
Section I

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	Total

Section II

17	18	19	20	21	22	23	24	Total

Grand Total

SECTION I (50 marks)

Answer all the questions in this section in the spaces provided.

1. Solve for n

$$\frac{6n}{n-1} = \frac{25}{n}.$$

(3 marks)

$n(6n) = 25(n-1)$

$6n^2 - 25n + 25 = 0$ ✓

$6n^2 - 15n - 10n + 25 = 0$

$(3n-5)(2n-5) = 0$ ✓

$3n - 5 = 0 \Rightarrow n = \frac{5}{3} = 1\frac{2}{3}$

or $2n - 5 = 0 \Rightarrow n = \frac{5}{2} = 2\frac{1}{2}$

} ✓ both

2. A family used two-fifths of its monthly income on school fees. Three-quarters of the remaining amount was used on family upkeep while the rest was invested. The family invested Ksh 13 500 monthly.

Calculate the amount of money the family used on school fees every month. (4 marks)

Fees $= \frac{2}{5}$; Rem. $= \frac{5}{5} - \frac{2}{5} = \frac{3}{5}$ ✓

upkeep $= \frac{3}{4} \times \frac{3}{5} = \frac{9}{20}$; Rem. $= \frac{3}{5} - \frac{9}{20} = \frac{3}{20}$ ✓

Invest. $= \frac{3}{20} = 13500$

$\frac{2}{5} = \frac{2}{5} \times 13500 \times \frac{20}{3}$ ✓

$=$ Ksh. 36 000 ✓

3. Solve for x in the equation.

$5^{2x-1} - 25^x = 500$ (3 marks)

$5^{2x-1} - 5^{2x} = 500$

$5^{2x} \cdot 5^{-1} - 5^{2x} = 500$ ✓

Let 5^{2x} be k

$\therefore \frac{1}{5}k - k = 500$

$-\frac{4}{5}k = 500$

$k = 500 \times \frac{5}{-4} = -625$

$\Rightarrow 5^{2x} = -625$ ✓

$5^{2x} = -5^4$ ✓

x is indeterminate / complex number

4. Kipkoech and Tanui began a 5 000 m race together at the starting line. Kipkoech and Tanui took 72 seconds and 80 seconds respectively to run a 400 m lap. The two athletes were together again at the starting line after some time.

Determine the number of laps that Tanui had to run to complete the race after they were together. (3 marks)

2	72	80
2	36	40
2	18	20
2	9	10
3	9	5
3	3	5
5	1	5
	1	1

Time taken to be together again:

LCM $= 2^4 \times 3^2 \times 5 = 720$ sec. ✓

No. of laps made by Tanui $= \frac{720}{80} = 9$

Remaining laps $= \frac{5000}{400} - 9$ ✓ $= 3\frac{1}{2}$ laps. ✓

5. Simplify

$$\frac{18ax - (3a-4x)(3a+4x)}{3a-8x}$$ (3 marks)

$(3a-4x)(3a+4x) = 9a^2 - 16x^2$

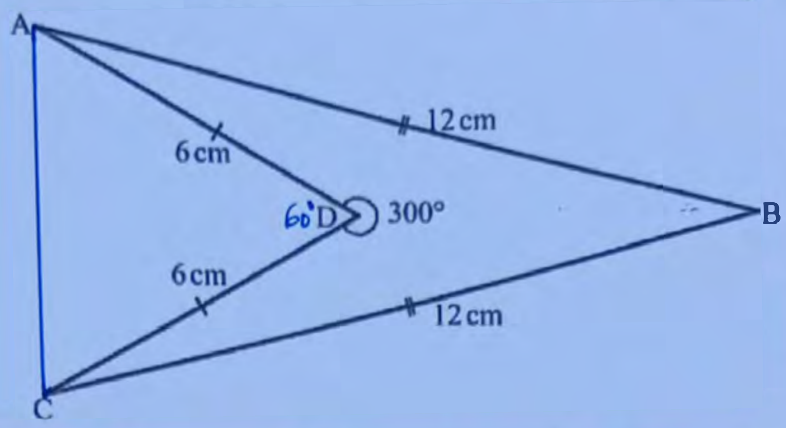
$$\frac{18ax - (9a^2 - 16x^2)}{3a-8x} = \frac{16x^2 + 18ax - 9a^2}{3a-8x}$$

$$= \frac{16x^2 + 24ax - 6ax - 9a^2}{3a-8x}$$

$$= \frac{(2x+3a)\cancel{(8x-3a)}}{-1\cancel{(8x-3a)}}$$ ✓

$= -2x - 3a$ ✓

6. In the quadrilateral ABCD, AD = CD = 6 cm and BA = BC = 12 cm. Angle ADC = 300°.



Calculate, correct to 2 decimal places, the area of the quadrilateral ABCD. (4 marks)

A. of $\Delta ADC = \frac{1}{2} \times 6 \times 6 \sin 60 = 9\sqrt{3}$ cm²

LSF $= \frac{12}{6} = 2 \Rightarrow$ ASf $= 2^2 = 4$

A. of $\Delta ABC = 4 \times 9\sqrt{3} = 36\sqrt{3}$ cm²

A. of ABCD $= 36\sqrt{3} - 9\sqrt{3} = 27\sqrt{3}$ cm²

$= 46.77$ cm²

7. A watch loses 8 seconds every hour. It was set to read the correct time at 1100 h on Sunday.

Determine the time, in a 12-hour system, the watch will show on the following Thursday when the correct time is 0500 h. (3 marks)

1100h Sun. ⟶ 1100h Wed. $= 24 \times 3 = 72$h.

1100h Wed. ⟶ 0500h Thur. $=$ 18h.

Total $= 18 + 72 = 90$h.

Time lost $= \frac{90 \times 8}{60} = 12$ min.

Time on Thur $=$ 5:00 a.m. $-$ 12 min

$=$ 4:48 a.m.

8. A lorry left town A for town B and maintained an average speed of 50 km/h. A car left town A for town B 42 minutes later and maintained an average speed of 80 km/h. At the time the car arrived in town B, the lorry had 25 km to cover to town B.

Determine the distance between town A and B. (3 marks)

A |—— x km ——| B

Time taken by car $= \frac{x}{80}$ h.

Distance by lorry when car arrived at B $= 50 \times \frac{42}{60} + 50\times\left(\frac{x}{80}\right) = (x-25)$

$$0.625x + 35 = x - 25$$

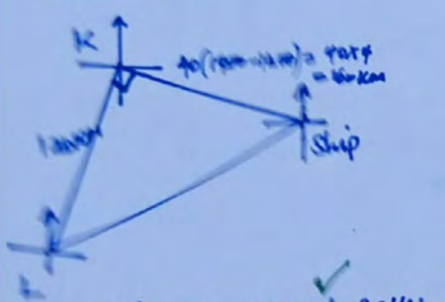
$$(1-0.625)x = 35+25$$

$$x = 160 \text{ km}$$

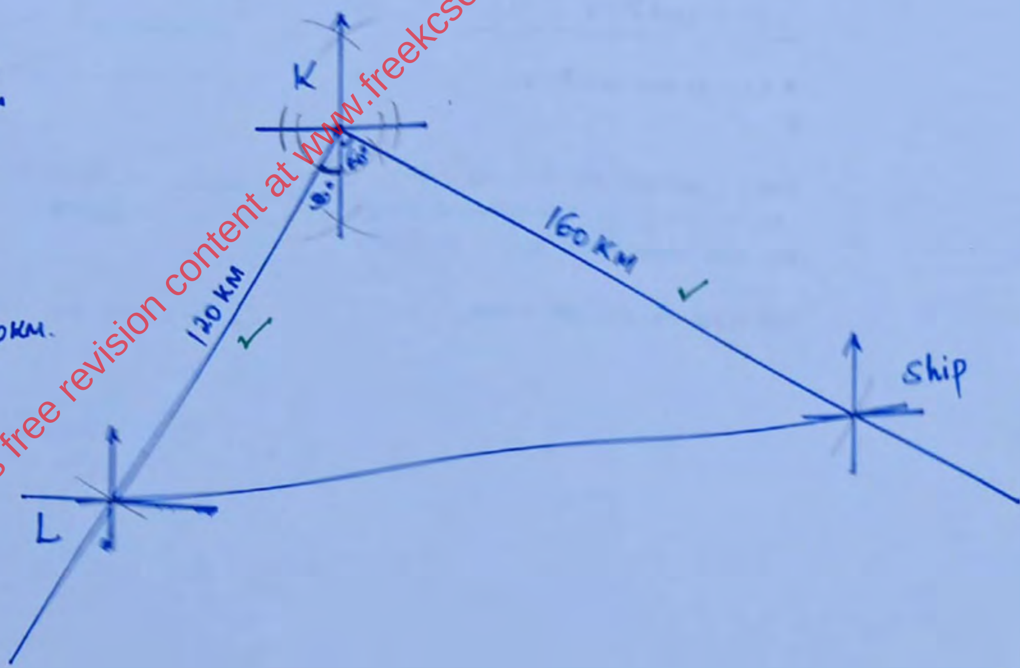
9. Port L is 120 km on a bearing of S30°W from port K. A ship left port K at 1000 h and sailed at a speed of 40 km/h along the bearing of S60°E.

Using scale drawing, determine the bearing of the ship from port L at 1400 h. (4 marks)

Sketch



Scale: 1cm represents 20KM.



Bearing is N 83° E

or 083°

10. The image of P(−2, 5) under a translation T is P′(2, 2). Q′(9, −5) is the image of Q under the same translation T.

Determine the coordinates of Q. (3 marks)

Let $\underset{\sim}{T} = \begin{pmatrix} x \\ y \end{pmatrix}$

$\therefore \begin{pmatrix} x \\ y \end{pmatrix} + \begin{pmatrix} -2 \\ 5 \end{pmatrix} = \begin{pmatrix} 2 \\ 2 \end{pmatrix}$ ✓

$x - 2 = 2 \Rightarrow x = 4$

$y + 5 = 2 \Rightarrow y = -3$

$\underset{\sim}{T} = \begin{pmatrix} 4 \\ -3 \end{pmatrix}$; Let $Q(a, b)$

$\Rightarrow \begin{pmatrix} 4 \\ -3 \end{pmatrix} + \begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} 9 \\ -5 \end{pmatrix}$ ✓

$4 + a = 9 \Rightarrow a = 5$

$-3 + b = -5 \Rightarrow b = -2$

$\therefore Q(5, -2)$ ✓

11. A Kenyan bank bought and sold United Arab Emirates (UAE) dirhams on two different dates as shown below.

		Buying (Ksh)	Selling (Ksh)
1st August 2021	1 UAE dirham	28.40	28.90
16th August 2021	1 UAE dirham	28.00	28.40

A Kenyan tourist who travelled to UAE on 1st August 2021 converted Ksh 130 050 to UAE dirhams.

During her stay in UAE, she spent 3520 UAE dirhams. She arrived back to Kenya on 16th August 2021. On the same day she converted the remaining amount of money to Kenya shillings at the same bank.

Calculate the amount of money in Kenya shillings that she received from the bank. (3 marks)

Dirhams received $= \dfrac{130\,050}{28.90}$ ✓ or 4 500 Dirhams

Remainder after expenses:

$= 4500 - 3520$ or 980 Dirhams

Ksh received $= 980 \times 28.00$ ✓

$=$ Ksh. 27 440 ✓

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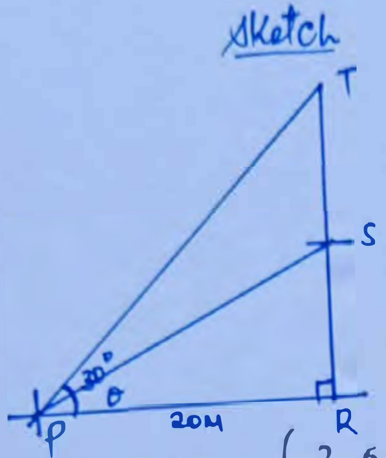
12. An electric post erected vertically is 20 m from point P on the same level ground. The angle of elevation of the top, T, of the post from P is 30°. Given that S is the mid point of the post, calculate, correct to 1 decimal place, the angle of elevation of S from P. (3 marks)

Sketch



$RT = 20\tan 30°$ or $11.55M$

$RS = \frac{1}{2} \times 20\tan 30°$ ✓

$= 10\tan 30°$ or $5.774M$

$\theta = \tan^{-1}\left(\frac{10\tan 30°}{20}\right)$ ✓ or $\tan^{-1}\left(\frac{5.774}{20}\right)$

$= 16.1°$ ✓

13. Given that $A = \begin{pmatrix} 2 & 6 \\ 2u & 5 \end{pmatrix}$, $B = \begin{pmatrix} 7 & -3 \\ -u & 5 \end{pmatrix}$ and $BA = \begin{pmatrix} 2 & v \\ 16 & w \end{pmatrix}$, determine the values of u, v and w. (3 marks)

$$\underset{\sim}{BA} = \begin{pmatrix} 7 & -3 \\ -u & 5 \end{pmatrix}\begin{pmatrix} 2 & 6 \\ 2u & 5 \end{pmatrix} = \begin{pmatrix} 7\times 2 + {}^{-}3\times 2u & 7\times 6 + {}^{-}3\times 5 \\ -u\times 2 + 5\times 2u & -u\times 6 + 5\times 5 \end{pmatrix} = \begin{pmatrix} 14-6u & 27 \\ 8u & 25-6u \end{pmatrix} ✓$$

$$\therefore \begin{pmatrix} 14-6u & 27 \\ 8u & 25-6u \end{pmatrix} = \begin{pmatrix} 2 & v \\ 16 & w \end{pmatrix} ✓$$

$\Rightarrow 8u = 16 \Rightarrow u = 2$

$\Rightarrow v = 27$ } ✓ for all

$\Rightarrow 25 - 6(2) = w \Rightarrow w = 13$

14. The capacities of two similar containers are 54 ml and 250 ml respectively. The difference in the heights of the two containers is 4 cm.

Calculate the height of the larger container. (3 marks)

$VSF = \frac{250}{54} = \frac{125}{27}$

$LSF = \left(\frac{125}{27}\right)^{1/3} = \left(\frac{5}{3}\right)^{3 \times 1/3} = 5/3$

$\therefore \; 5/3 = \frac{x+4}{x}$

$5x = 3x + 12 \Rightarrow x = 6$

H. of larger container $= 6 + 4 = 10$ cm

15. The table below shows the mean marks in a mathematics test of two classes.

Class	Number of students	Mean mark
X	43	65
Y	45	62

Calculate, correct to 2 decimal places, the mean mark of the classes. (2 marks)

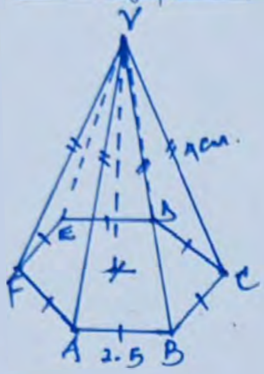
$\text{Mean} = \frac{(43 \times 65) + (45 \times 62)}{(43 + 45)}$

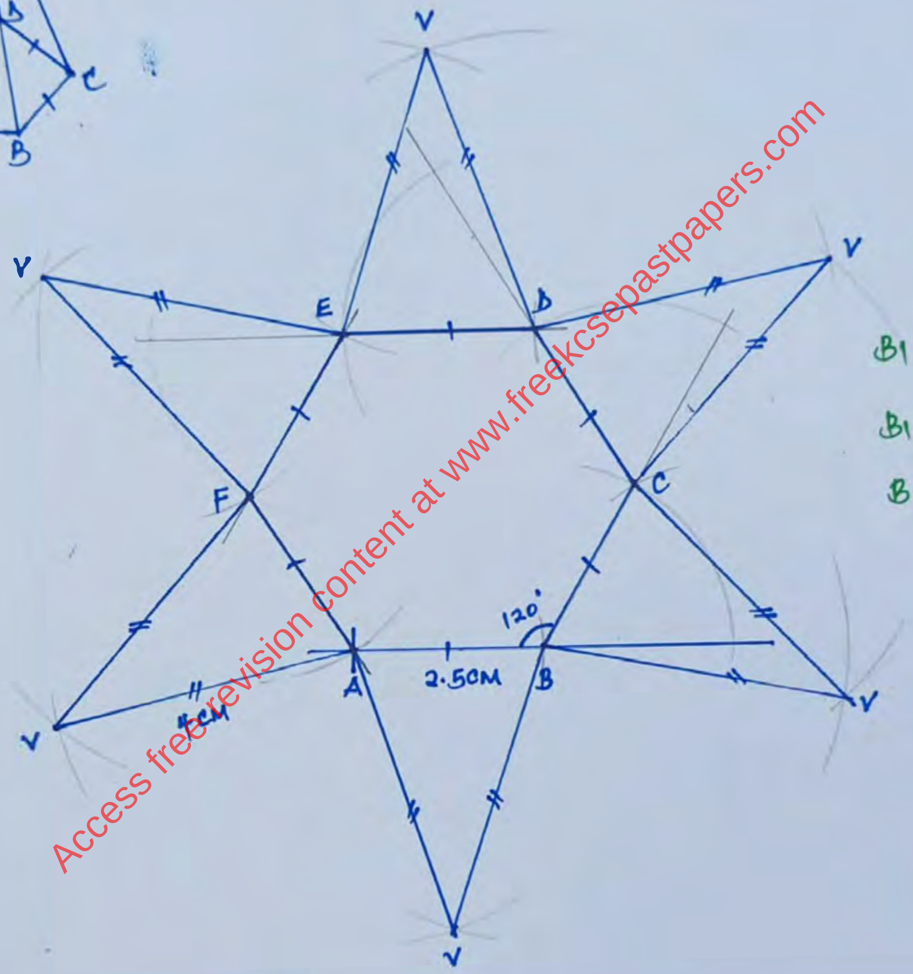
$= 63.47$

16. The base, ABCDEF, of a right pyramid is a regular hexagon of side 2.5 cm. Point V is the vertex of the pyramid and the length of the slanting edges is 4 cm.

Draw a labelled net of the pyramid. (3 marks)

Sketch of pyramid





B1 ✓ Hexagon

B1 ✓ Isosceles Δs

B1 ✓ Labelling

SECTION II (50 marks)

Answer only five questions in this section in the spaces provided.

17. A contractor hired Wema and Tatu to transport 144 tonnes of stones to building sites A and B.

To transport 48 tonnes of stones for a distance of 28 km, the contractor paid Ksh 24 000.

(a) Wema transported 96 tonnes of stones to site A, a distance of 49 km.

(i) Calculate the amount of money that was paid to Wema. (2 marks)

$\frac{49}{28} \times \frac{96}{48} \times 24\,000$ ✓

Ksh. 84 000 ✓

(ii) For every 8 tonnes of stones Wema transported to site A, he spent Ksh 3 000.

Calculate the profit Wema made. (3 marks)

Expenses $= \frac{96}{8} \times 3000 = 36\,000$ ✓

Profit $= 84\,000 - 36\,000$ ✓

$=$ Ksh. 48 000 ✓

(b) Tatu transported the remaining 48 tonnes of stones to site B, a distance of 84 km. If Tatu made 44% profit, calculate the amount of money Tatu spent to transport the stones. (3 marks)

Paid: $\frac{84}{28} \times \frac{48}{48} \times 24\,000$ ✓ $=$ Ksh. 72 000

Expenses $= \frac{100-44}{100} \times 72\,000$ ✓

$=$ Ksh. 40 320 ✓

(c) Determine the ratio of the profit made by Wema to that made by Tatu. (2 marks)

$\frac{44}{100} \times 72\,000 =$ Ksh. 31 680

W : T $= 48\,000 : 31\,680$ ✓

$= 50 : 33$ ✓

18. A shot put is spherical and has mass of 7.26 kg. It is made of a metal with a density of 6.93 g/cm³.

(Take $\pi = \frac{22}{7}$).

(a) Determine the radius of the shot put, correct to 1 decimal place. (3 marks)

$$\text{Vol.} = \frac{4}{3} \times \frac{22}{7} \times r^3 = \frac{7.26 \times 1000}{6.93}$$

$$r^3 = \frac{7.26 \times 1000 \times 21}{6.93 \times 4 \times 22} = 250$$

$$r = \sqrt[3]{250} \qquad = 6.3\text{ cm}$$

(b) A bucket is in the shape of a frustum of a cone. The base radius of the bucket is 7 cm.

The bucket contains water to a height of 15 cm. The radius of the surface of the water is 10.5 cm.

(i) Find the volume of the water in the bucket. (3 marks)

$$\frac{10.5}{7} = \frac{h+15}{h} \Rightarrow h = 30\text{ cm}$$

$$H = 45\text{ cm}$$

$$V = \frac{1}{3} \times \frac{22}{7}\left(10.5^2 \times 45 - 7^2 \times 30\right)$$

$$= 3657.5\text{ cm}^3$$

*(ii) The shot put ball is completely submerged in the water in the bucket.

Calculate the new height of the water in the bucket. (4 marks)

$$\text{New Vol.} = 3657.5 + \frac{7.26 \times 1000}{6.93}$$

$$= \frac{197615}{42}\text{ cm}^3 \text{ or } 4705.119047 6\text{ cm}^3$$

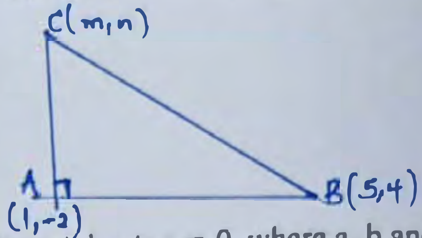
$$\text{VSF} = \left(\frac{45+x}{45}\right)^3 = \frac{4705.1190476}{3657.5}$$

$$x = 3.941$$

$$\text{New height} = 15 + 3.941$$

$$= 18.941\text{ cm.}$$

19. A triangle ABC is right angled at point A. The vertices of the triangle are A(1, −2), B(5, 4) and C(m, n).



The equation of line BC is $5y - x = 15$.

(a) Determine:

(i) the equation of line AC in the form $ax + by + c = 0$, where a, b and c are integers. (4 marks)

$5y = x + 15 \Rightarrow y = \frac{1}{5}x + 3$

Grad. of $\overline{AB} = \frac{4-(-2)}{5-1} = \frac{6}{4} = \frac{3}{2}$

Grad. of $\overline{AC} = \frac{-1}{\frac{3}{2}} = -\frac{2}{3}$

$\frac{y-(-2)}{x-1} = -\frac{2}{3}$

$\Rightarrow -2x - 3y - 4 = 0$ or $2x + 3y + 4 = 0$

(ii) the coordinates of point C. (3 marks)

BC: $(5y - x = 15) \times 2$

AC: $(3y + 2x = -4) \times 1$

$10y - 2x = 30$

$3y + 2x = -4$

$13y + 0 = 26$

$13y = 26 \Rightarrow y = 2$

$5(2) - x = 15 \Rightarrow x = -5$

$\Rightarrow m = x = -5$

$n = y = 2$

$\therefore C(-5, 2)$

(b) A line passes through point A and is parallel to line BC.

Determine the x-intercept of the line. (3 marks)

BC: $y = \frac{1}{5}x + 3 \Rightarrow m_1 = \frac{1}{5}$

$m_1 = m_2 = \frac{1}{5}$

$\frac{y-(-2)}{x-1} = \frac{1}{5}$

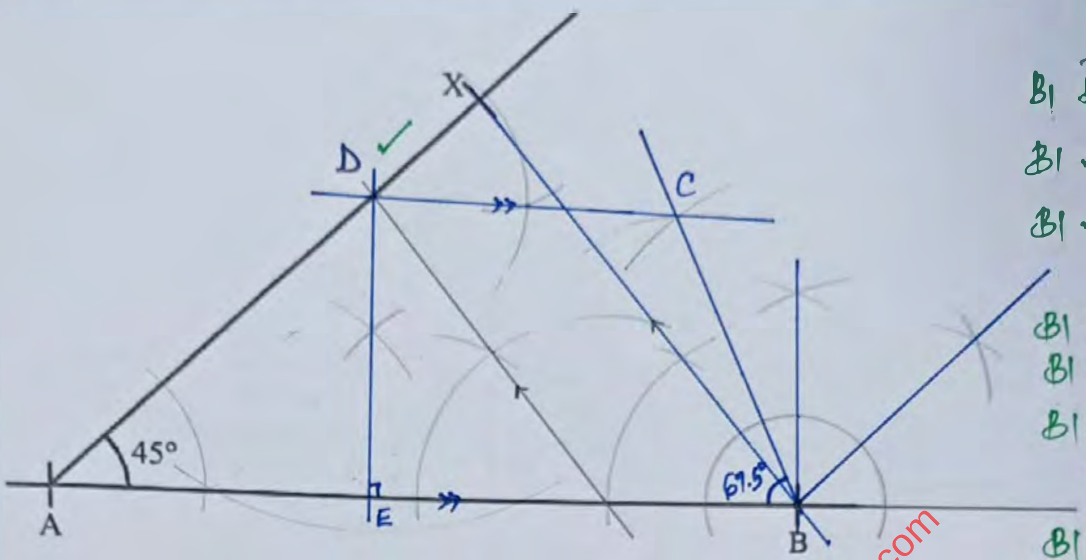
$y + 2 = \frac{1}{5}x - \frac{1}{5}$

$y = \frac{1}{5}x - 2\frac{1}{5} = 0$

$x = 11$

x- intercept $= 11$

20. In the figure below, line AB = 10 cm and is part of a trapezium ABCD. Point X is such that angle BAX = 45°.



B1 join B to X

B1 ✓ arc on AB and parallel line to BX

B1 ✓ loc. of D

B1 Constr. ∠135° at B

B1 Bisect ∠135° at B

B1 loc. of C and line DC

B1 ✓ DE constructed

(a) Using a ruler and a pair of compasses only:

(i) locate point D on line AX such that AD : DX = 3:1. (3 marks)

(ii) complete trapezium ABCD such that line DC is parallel to line AB and angle ABC = 67.5°. (3 marks)

(iii) draw a perpendicular line from D to meet AB at E. Measure DE. (2 marks)

DE = 4.0 cm ✓ ±0.1 cm

(b) Calculate the area of the trapezium ABCD. (2 marks)

$A = \frac{1}{2} \times DE(AB + CD)$

DE = 4 cm, AB = 10 cm, CD = 4 cm

$= \frac{1}{2} \times 4(10 + 4)$ ✓

$= 28\ cm^2$ ✓

21. The amount of money, in Kenya shillings, spent on airtime by a group of 30 people in a period of an hour was recorded as shown below.

27	20	21	24	22	25
42	34	55	26	30	39
35	46	32	21	38	34
31	37	27	29	32	56
23	44	25	31	28	30

(a) Complete the frequency distribution table below.

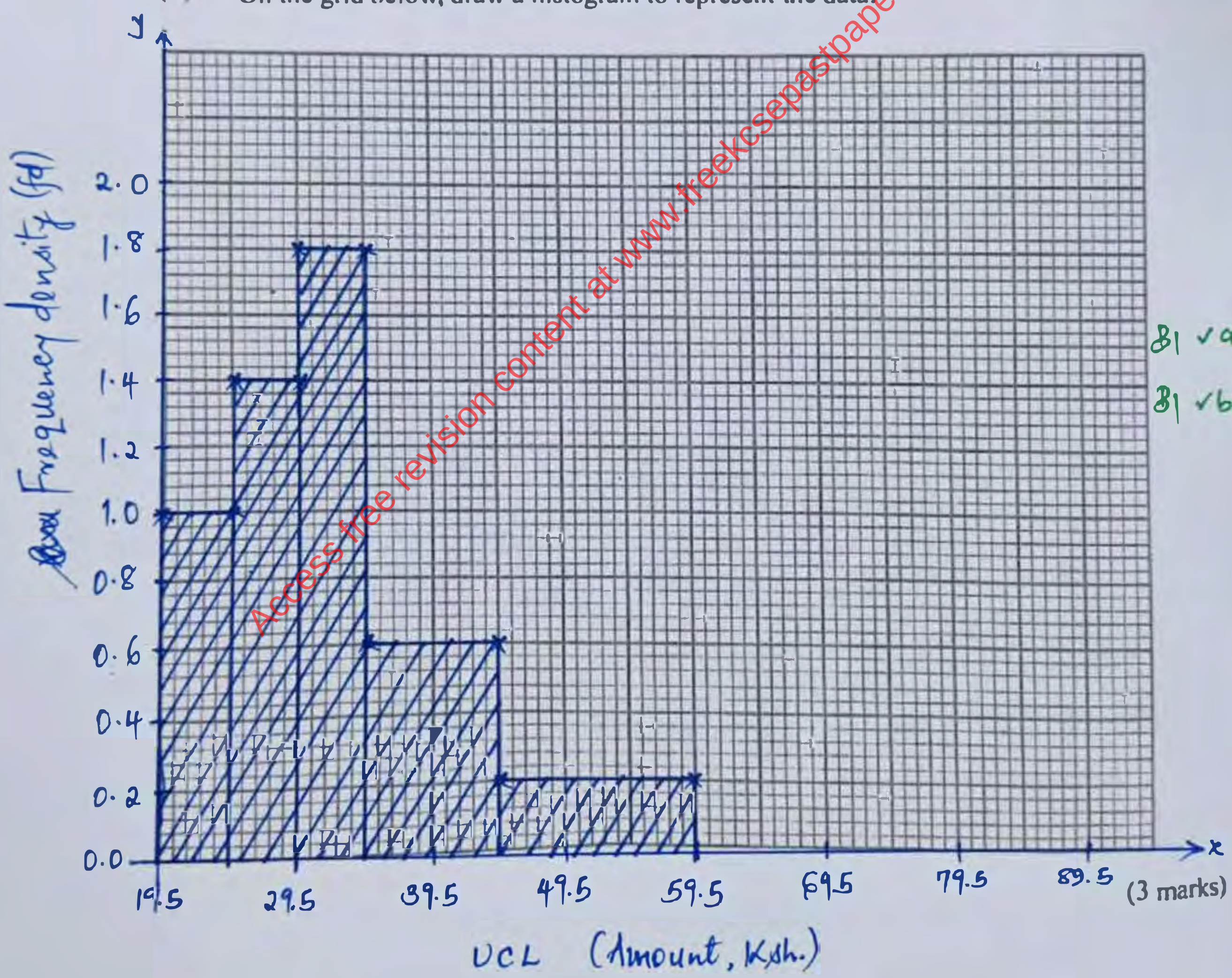
Tally	HHT	HHT II	HHT IIII	HHT I	III
Amount (Ksh)	20 – 24	25 – 29	30 – 34	35 – 44	45 – 59
Frequency	5	7	9	6	3
fd	1	1.4	1.8	0.6	0.2

B2 for all 5 values ✓

B1 for fd

(2 marks)

(b) On the grid below, draw a histogram to represent the data.



B1 ✓ axes

B1 ✓ bars

(3 marks)

(c) Use the histogram to determine:

(i) the median amount of money spent on airtime by the 30 people. **(3 marks)**

$A = 5\times 1.0 + 5\times 1.4 + 1.8x = \frac{1}{2}\times 30$ ✓

$1.8x = 15 - 12 = 3$

$x = 1\frac{2}{3}$ or 1.667 ✓

Median $= 29.5 + 1.667$

$= 31.167$ ✓

≈ 31.17

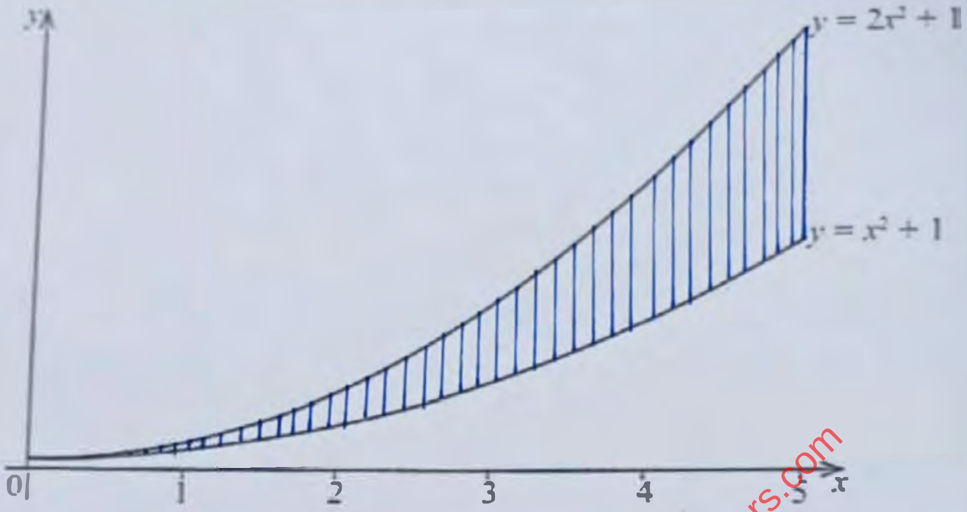
(ii) the number of people who spent more than Ksh 41.50 on airtime over that period. **(2 marks)**

$5\times 1.0 + 5\times 1.4 + 5\times 1.8 + 7\times 0.6$ ✓

$= 25.2$

≈ 26 people. ✓

22. The diagram below is a sketch of two curves $y = 2x^2 + 1$ and $y = x^2 + 1$ drawn on the same grid.



(a) Using the trapezium rule with 5 strips, estimate the area bounded by the curves $y = 2x^2 + 1$, $y = x^2 + 1$ and the lines $x = 0$ and $x = 5$. (5 marks)

X	0	1	2	3	4	5
$2x^2+1 = y_1$	1	3	9	19	33	51
$x^2+1 = y_2$	1	2	5	10	17	26
$y = (y_1 - y_2)$	0	1	4	9	16	25

$A = \frac{1}{2}\{(0+25) + 2(1+4+9+16)\}$ ✓

$= 42\frac{1}{2}$ sq. units ✓

(b) Using the mid ordinate rule with 5 strips, estimate the area bounded by the curves $y = 2x^2 + 1$, $y = x^2 + 1$ and the lines $x = 0$ and $x = 5$. (5 marks)

X	0.5	1.5	2.5	3.5	4.5	
$2x^2+1 = y_1$	1.5	5.5	13.5	25.5	41.5	✓
$x^2+1 = y_2$	1.25	3.25	7.25	13.25	21.25	✓
$(y_1 - y_2) = y$	0.25	2.25	6.25	12.25	20.25	✓

$A = 1(0.25 + 2.25 + 6.25 + 12.25 + 20.25)$ ✓

$= 41\frac{1}{4}$ sq. units ✓

23. A Supermarket sold 530 packets of milk daily when the price was Ksh 50 per packet.

Whenever the price per packet was increased by Ksh 4, the number of packets sold daily decreased by 20.

If n represents the number of times the price was increased:

(a) write an expression in terms of n for:

(i) the price of a packet of milk after the price was increased. (1 mark)

$= (50 + 4n)$ ✓

(ii) the number of packets of milk sold after the price was increased. (1 mark)

$= (530 - 20n)$ ✓

(iii) the total sales, in simplified expanded form, after the price of a packet of milk was increased. (2 marks)

$S = (50 + 4n)(530 - 20n)$ ✓

$= -80n^2 + 1120n + 26\,500$

$S = -80n^2 + 1120n + 26\,500$ ✓

(b) Determine:

(i) the number of times the price was increased to attain maximum sales. (3 marks)

$\frac{dS}{dn} = 0 = -160n + 1120$ ✓✓

$n = 7$ ✓

(ii) the price of a packet of milk for maximum sales. (1 mark)

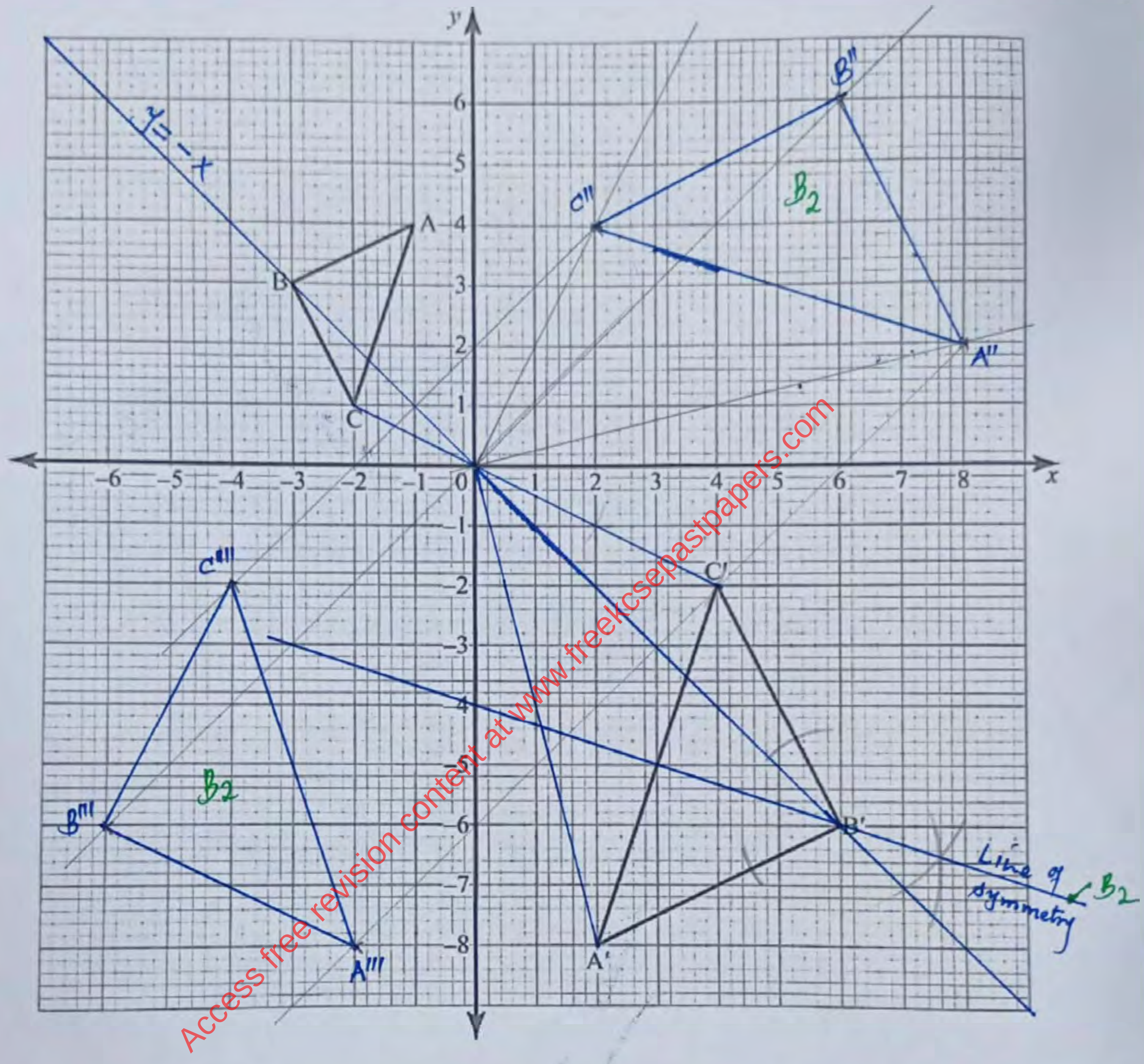
$50 + 4 \times 7 = 78$ Ksh. ✓

(iii) the maximum sales. (2 marks)

$S = -80(7)^2 + 1120(7) + 26\,500$ ✓

$=$ Ksh. $30\,420$ ✓

24. Triangle ABC and A'B'C' are drawn on the grid provided.



(a) Describe fully a single transformation that mapped triangle ABC onto triangle A'B'C'. (2 marks)

Enlargement, scale factor −2 and centre (0,0)

(b) On the same grid, draw:

(i) triangle A″B″C″ the image of triangle A′B′C′ under a rotation of +90° about O (0, 0). (2 marks)

A″(8, 2), B″(6, 6), C″(2, 4)

with evidence

(ii) triangle A‴B‴C‴, the image of triangle A″B″C″ under a reflection in the line $y = -x$. (2 marks)

A‴(−2, −8), B‴(−6, −6), C‴(−4, −2)

with evidence

(c) Draw the line of symmetry of triangle A′B′C′ and hence determine its equation in the form $y = mx + c$, where m and c are constants. (4 marks)

(6, −6), (3, −5)

$m = \frac{-5-(-6)}{3-6} = -\frac{1}{3}$; $c = -4$ } ✓ for ✓M and ✓C

$\therefore y = -\frac{1}{3}x - 4$ ✓

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