Name: $\qquad$
$\qquad$ Stream $\qquad$
Date $\qquad$ Sign $\qquad$
121/2
Mathematics Alt. A
May 2023
Form Four
Time: $21 ⁄ 2$ Hours

# CHANIA HIGH SCHOOL FORM FOUR SYLLABUS EXAMINATIONS, 2023 

## Kenya Certificate Of Secondary Education

Mathematics Alt. A
121/2
Time: $21 / 2$ Hours

## INSTRUCTIONS TO CANDIDATES;

(a) Write your Name, Admission Number and Class in the spaces provided.
(b) This paper contains Section I and II.
(c ) ` Attempt ALL questions in Section I and any FIVE in Section II.
(d) Show all your workings in the spaces provided below each question.
(e) Marks may be given for correct working even if the answer is wrong.
(f) Use Electronic Calculator and Mathematical Tables where possible.
(g) Candidates must check the question paper to ascertain that no pages are missing.

FOR EXAMINERS USE ONLY

## SECTION I

| $\mathbf{1}$ | $\mathbf{2}$ | $\mathbf{3}$ | $\mathbf{4}$ | $\mathbf{5}$ | $\mathbf{6}$ | $\mathbf{7}$ | $\mathbf{8}$ | $\mathbf{9}$ | $\mathbf{1 0}$ | $\mathbf{1 1}$ | $\mathbf{1 2}$ | $\mathbf{1 3}$ | $\mathbf{1 4}$ | $\mathbf{1 5}$ | $\mathbf{1 6}$ | TOTAL |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |

SECTION II

| 17 | 18 | 19 | 20 | 21 | 22 | 23 | 24 | 25 | TOTAL |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| GRAND |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |
| TOATAL |  |  |  |  |  |  |  |  |  |



Section I: (50marks)
1.Under a transformation whose matrix is $\left(\begin{array}{cc}x-1 & 3 \\ 1 & x+1\end{array}\right)$, an object of area $12 \mathrm{~cm}^{2}$ is mapped onto an image whose area is $60 \mathrm{~cm}^{2}$. Find the possible values of $x$.
2.Solve for $x$ in the equation

$$
-3 \sin ^{2} x+8 \cos x=0 \text { for } 0^{0} \leq x \leq 360^{\circ}
$$

3. Use the trapezoidal rule to approximate the area bounded by the curve $y=-x^{2}-3 x+10$ and the x -axis by using 4 trapezoids of equal width from $x=-4$ to $x=0$
(3 marks)
4. A point $\mathrm{M}\left(60^{\circ} \mathrm{N}, 18^{\circ} \mathrm{E}\right)$ is on the surface of the earth. Another point N is situated at a distance of 630 nautical miles east of M.Find
a) the longitude difference between M and N
\{2 marks \}
b) the position of N .
5.The equation of curve; $y=x^{3}+2 x+1$. Find the equation of the tangent to the curve at point $\mathrm{x}=1$.
5. The figure below represents a cuboid $P Q R S T U V W . ~ P Q=60 \mathrm{~cm}, \mathrm{QR}=11 \mathrm{~cm}$ and $\mathrm{RW}=10 \mathrm{~cm}$.


Calculate the angle between line PW and plane PQRS, correct to 2 decimal places.
7. Calculate the standard deviation for the following set of data (use actual mean)
8. Draw a line $P Q=4 \mathrm{~cm}$. Indicate by shading the region within which a variable point must lie if $P A \leq 3 \mathrm{~cm}$ and $P A>A Q$.
(3 marks)
9. Complete the table below for the function $y=3 x^{2}-8 x+10$

| $x$ | 0 | 2 | 4 | 6 | 8 | 10 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $y$ | 10 | 6 | 26 |  | 138 |  |

Hence estimate the area bounded by the curve $y=3 x^{2}-8 x+10$ and the lines $y=0, x=0$ and $x=10$ using trapezoidal rule with 5 strips.
(3marks)
10. Evaluate $\int_{-1}^{2}\left(2 x^{2}-3 x-14\right) d x$
11. Find the area under curve $y=x^{2}+2$, between $\mathrm{x}=2$ and $\mathrm{x}=6$ by trapezium rule $\operatorname{sing} 4$ strips.
(3 marks)
12. Solve for $\vartheta$ in the equation.

$$
\operatorname{Sin}(2 \theta-10)=-0.5 \text { for } 0^{0} \leq \theta \leq 360^{\circ}
$$

13. The table below shows the number of defective bolts from a sample of 40

| No of bolts | 0 | 1 | 2 | 3 | 4 | 5 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| Frequency | 20 | 8 | 6 | 4 | 1 | 1 |

Calculate the standard deviation of the data above.
14. State the amplitude, period and phase angle of $y=2 \sin \left(\frac{1}{2} x+30^{\circ}\right)$
(i) Amplitude
(1 mark)
(ii) Period
(iii) Phase angle
15. The marks obtained by 10 students in a maths test were:-

## $25,24,22,23, x, 26,21,23,22$ and 27

The sum of the squares of the marks, $\Sigma x^{2}=5154$
Calculate the:
(a) value of $\boldsymbol{x}$
(2mks)
(b) Standard deviation
16. Solve the equation $4-4 \cos ^{2} \mathrm{x}=4 \operatorname{Sin} \mathrm{x}-1$ for the range $0^{\circ} \leq x \leq 360$

## SECTION B (50 MARKS)

Answer any five questions in this section
17.Two towns $P$ and $Q$ lie on the same parallel latitude such that $P$ is due east of $Q$. When local time at $Q$ is 9.50 am , the local time at P is 3.12 pm .
(a) Find the latitude difference between P and Q .
(b) Give that the longitude of P is $52^{\circ} \mathrm{E}$, find the longitude of Q .

## (2 marks)

(c)A pilot took off from town Q and flew to town P along the parallel of latitude. The pilot took $31 / 4$ hours travelling at an average speed of $860 \mathrm{~km} / \mathrm{h}$ to reach $P$. Calculate to $1 \mathrm{~d} . \mathrm{p}$ the latitude of town $P$ and $Q$ if they both lie in the northern hemisphere.
(3 marks)
(d)Two towns $R$ and $S$ are both on the equator and 3820 nm apart. Town $R$ is due west of town $S$. Find the local time at R when the local time at S is 2.20 pm . (Take $\mathrm{R}=6370 \mathrm{~km}$ and $\pi=\frac{22}{7}$ )
(3 marks)
18.The marks of 50 students in a mathematics test were taken from a form 4 class and recorded in the table below

| Marks | $21-30$ | $31-40$ | $41-50$ | $51-60$ | $61-70$ | $71-80$ | $81-90$ | $91-100$ |
| :--- | :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| frequency | 2 | 5 | 7 | 9 | 11 | 8 | 5 | 3 |

a) On the grid provided, draw a cumulative frequency curve of the data, using 1 cm to represent 5 students and 1 cm to represent 10 marks.
b) From your curve in (a) above;
i) Estimate the median mark
ii) Determine the interquartile deviation
iii) Determine the $10^{\text {th }}$ to $90^{\text {th }}$ percentile range.
c ) It is given that students who score over 45 marks pass the test. Use your graph in (a) above to estimate the percentage of students that pass.
19. The diagram below shows a straight line $2 x+y=8$ intersecting the curve $y=2 x^{2}-4 x+4$ at the points P and Q .

(a) Find the coordinates of P and Q .
(b) Calculate the area of the shaded region.
(4 marks)
(c) Find the coordinates of the stationary points on the curve $y=2 x^{2}-4 x+4$ (3 marks)
20.


The above diagram represents a wooden prism. ABCD is a rectangle. Points E and F are directly below $C$ and $B$ respectively. $M$ is the midpoint of $C D . A B=8 \mathrm{~cm}, B C=10 \mathrm{~cm}$ and $C E=4.5 \mathrm{~cm}$.
a) The size of angle CDE.
b) Calculate:
i) Length of AC.
ii) The angle CAE makes with the plane ADEF.
c) Find the:
i) Length of MB.
ii) Angle CBM.
21. Complete the table below, giving the values correct to 1 decimal place.

| $\mathrm{x}^{\circ}$ | 0 | 40 | 80 | 120 | 160 | 200 | 240 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $2 \sin (\mathrm{x}+$ <br> $20)^{\circ}$ | 0.7 |  | 2.0 |  | 0.0 |  | -2.0 |
| $\sqrt{3} \cos \mathrm{x}$ | 1.7 | 1.3 |  | -0.9 |  | -1.6 |  |

b) On the grid provided, using the same scale and axes, draw the graphs of $y=2 \sin (x+20)^{\circ}$ and $\mathrm{y}=\sqrt{3} \cos \mathrm{x}$ for $0^{\circ} \leq \mathrm{x} \leq 240^{\circ}$.
marks $\}$

c) Use the graphs drawn in (b) above to determine:
i) the values of $x$ for which $2 \sin (x+20)=\sqrt{3} \cos x$
22. a) A triangular garden ABC is such that $\mathrm{AB}=8 \mathrm{~cm},\left\llcorner\mathrm{BAC}=45^{\circ}\right.$ and $\left\llcorner\mathrm{ABC}=75^{\circ}\right.$. Using an appropriate scale draw the garden using a ruler and pair of compasses only
b) A water tap $P$ is to be mounted in the garden such that it is equal in distance from $A, B$ and C. on the diagram in (a) show he position of point P .
a) A section of the plot is enclosed such that a region $R$ is formed under the following conditions.
i) $\quad \mathrm{CR} \geq 1.5 \mathrm{~m}$
(2 marks)
ii) $\quad \mathrm{R}$ is more than 2 m from line AB
(1 mark)
iii) $\quad \mathrm{R}$ is nearer to CB than CA . Shade the region R formed
23. The displacement $S$ metres of a particle from a fixed point in motion at any given time ( t ) seconds is given by $s=3 t+3 / 2 t^{2}-2 t^{3}$.
a) Find the initial acceleration. (3 marks)
b) Calculate
i) The time when the particle was momentarily at rest
ii) Its displacement by the time it comes to rest momentarily
(2 marks)
iii) Calculate the maximum velocity attained (3 marks)
24. A firm has a fleet of vans and trucks. Each van can carry 9 crates and 3 cartons. Each truck can carry 4 crates and 10 cartons. The firm has to deliver not more than 36 crates and at least 30 cartons.
(a) If $x$ vans and $y$ trucks are available to make the delivery. Write down inequalities to represent the above information.
(b) Use the grid provided, to represent the inequalities in (a) above

(c) Given that the cost of using a truck is four times that of using a van, determine the number of vehicles that may give minimum cost
(2 Marks)
25. The marks of 50 students in a mathematics test were taken from a form 4 class and recorded in the table below.

| Mark (\%) | $21-30$ | $31-40$ | $41-50$ | $51-60$ | $61-70$ | $71-80$ | $81-90$ | $91-100$ |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| Frequency | 2 | 5 | 7 | 9 | 11 | 8 | 5 | 3 |

(a) On the grid provided, draw a cumulative frequency curve of the data.
(3marks)

(b) From your curve in (a) above
(i) Estimate the median mark.
(ii) Determine the Interquartile deviation.
(iii) Determine the $10^{\text {th }}$ to $90^{\text {th }}$ percentile range.
(c) It is given that students who score over 45 marks pass the test. Use graph in (a) above to estimate the percentage of students that pass.
26. A triangle $A B C$ with vertices at $A(1,-1) B(3,-1)$ and $C(1,3)$ is mapped onto triangle $A^{1} B^{1} C^{1}$ by a transformation whose matrix is $\left(\begin{array}{ll}-1 & 0 \\ 0 & 1\end{array}\right)$

Triangle $A^{1} B^{1} C^{1}$ is then mapped onto $A^{11} B^{11} C^{11}$ with vertices at $A^{11}(2,2) B^{11}(6,2)$ and $C^{11}$ $(2,-6)$ by a second transformation.
(i) Find the coordinates of $\mathrm{A}^{1} \mathrm{~B}^{1} \mathrm{C}^{1}$
(ii) Find the matrix which maps $\mathrm{A}^{1} \mathrm{~B}^{1} \mathrm{C}^{1}$ onto $\mathrm{A}^{11} \mathrm{~B}^{11} \mathrm{C}^{11}$.
(iii) Determine the ratio of the area of triangle $\mathrm{A}^{1} \mathrm{~B}^{1} \mathrm{C}^{1}$ to triangle $\mathrm{A}^{11} \mathrm{~B}^{11} \mathrm{C}^{11}$.
(iv) Find the transformation matrix which maps $\mathrm{A}^{11} \mathrm{~B}^{11} \mathrm{C}^{11}$ onto ABC

