## KERERI GIRLS' HIGH SCHOOL - 2023

Kenya Certificate of Secondary Education


# MATHEMATICS <br> -Alt. A <br> April. 2023-2 Hours 30 Mins 

Paper 2


Name:
Index Number: $\qquad$
Student's Signature:
Date: Class: $\qquad$

## Instructions to candidates

(i) Write your name, Index number and class in the spaces provided above.
(ii) Sign and write the date of examination in the spaces provided above.
(iii) This paper consists of two sections: Section I and Section II.
(iv) Answer all the questions in Section I and only five questions from Section II.
(v) Show all the steps in your calculations, giving your answers at each stage in the spaces provided below each question.
(vi) Marks may be given for correct working even if the answer is wrong.
(vii) Non - programmable silent electronic calculators and KNEC Mathematical tables may be used, except where stated otherwise.
(viii) This paper consists of 14 printed pages. Candidates should check the question paper to ascertain that all the pages are printed as indicated and that no questions are missing.
(ix) Candidates should answer the questions in English.

## For Examiner's Use Only

## Section I

| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | Total |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |

Section II

| 17 | 18 | 19 | 20 | 21 | 22 | 23 | 24 | Total |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
|  |  |  |  |  |  |  |  |  |

Grand Total

## SECTION I (50 Marks)

Answer all the questions in this section in the spaces provided.

1. Given that $9 x^{2}+p x+q$ is a perfect square and that $p-q=8$, find the values of $p$ and $q$.
2. The matrices $\mathbf{P}$ and $\mathbf{Q}$ are $\mathbf{P}=\left(\begin{array}{ll}x & 4 \\ 3 & 2\end{array}\right)$ and $\mathbf{Q}=\left(\begin{array}{ll}1 & 2 \\ 3 & 4\end{array}\right)$. Given that the determinant of $\mathbf{P Q}$ is 4 , find $x$.
3. Simplify $\frac{4}{3+\sqrt{5}}-\frac{\sqrt{5}}{3-\sqrt{5}}$ giving your answer in the form $\mathrm{a}+\mathrm{b} \sqrt{\mathrm{c}}$ where $\mathrm{a}, \mathrm{b}$ and c are real numbers.
4. The length and width of a rectangular sheet of paper measured to the nearest millimetre are 22.3 cm and 15.7 cm respectively. Calculate to four significant figures, the percentage error in area of the paper.
5. Make $K$ the subject of the formula $R=\frac{d}{2 \pi} \sqrt{\frac{F-h K}{K}}$ (3 marks)
6. Draw a line PQ of length 7 cm . On one side of the line PQ , construct the locus of a point R such that the area of triangle PRQ is $10.5 \mathrm{~cm}^{2}$. On this locus locate two positions of R , $\mathrm{R}_{1}$ and $\mathrm{R}_{2}$ such that $\angle \mathrm{PR}{ }_{1} \mathrm{Q}=\angle \mathrm{PR}_{2} \mathrm{Q}=90^{\circ}$.
7. Expand $\left(2 x-\frac{3}{x^{2}}\right)^{6}$ up to the constant term.

Hence use your expansion to estimate $(19.97)^{6}$.
8. Solve for $x$ in: $\log _{3} 4 x+3 \log _{27} x=-6$.
9. Determine the number of years it will take for an investment of Kshs. 2100000 to grow to Kshs. 3108513 if the interest rate is $16 \%$ per year compounded quarterly.
10. Given that O is the origin, $\mathbf{O A}=2 \mathbf{i}+2 \mathbf{j}-4 \mathbf{k}$ and $\mathbf{O B}=6 \mathbf{i}+10 \mathbf{j}+2 \mathbf{k}$. If R divides AB externally in the ratio 3:1. Find OR.
11. The second term of an Arithmetic Progression (A.P) is 10 while the fifth term of the same progression is 34 . Find the first term and the common difference.
12. The figure below shows a circle centre $O$, inscribed inside a quadrant centre $P$ and radius 14 cm . Calculate the value of $r$, radius of the circle.

13. The area of a triangle $A B C$ is $15 \mathrm{~cm}^{2}$. The triangle is transformed by the matrix $\left(\begin{array}{ll}2 & 5 \\ 4 & 1\end{array}\right)$ onto the image $A^{\prime} \mathrm{B}^{\prime} \mathrm{C}^{\prime}$. Calculate the area of the triangle $\mathrm{A}^{\prime} \mathrm{B}^{\prime} \mathrm{C}^{\prime}$.
(2 marks)
14. A cocoa blender makes a profit of $30 \%$ be selling a super mix cocoa at Kshs. 650 for 250 grams tin. He makes a super mix cocoa by blending two varieties of cocoa., A and B which cost him Kshs. 1800 and Kshs. 2400 per kg respectively. In what proportion were the cocoa types A and B mixed?
(3 marks)
15. Solve for $\theta$ in the equation $\tan (2 \theta-45)=-\frac{\sqrt{3}}{3}$ for $0^{\circ} \leq \theta \leq 360^{\circ}$.
16. A circle whose equation is given as $2 x^{2}+2 y^{2}-12 x+4 a y+2 a^{2}-32=0$ passes through a point $(3,1)$. Find the centre and the radius of the circle if $a$ is a negative integer. (4 marks)

SECTION II (50 Marks)
Answer any five questions from this section in the spaces provided.
17. Kisumu County government has a water reservoir at Kajulu Hills constructed in form of a cylinder with base radius of 8.4 metres and a height of 10 metres.
(a) Determine the capacity of the reservoir in litres.
(b) The tank is served by two inlet pipes A and B. A has a radius of 2.8 cm and water flows through it at a speed of $10 \mathrm{~m} / \mathrm{s}$ while B has a radius of 3.5 cm and water flows through it at a speed of $12 \mathrm{~m} / \mathrm{s}$. The reservoir is initially empty. Determine:
(i) The amount of water in litres in the tank after 2 hours.
(ii) The time it takes to fill the tank to the nearest minute.
(c) If an outlet pipe C has a radius of 4.2 cm and water flows through it at a speed of $10 \mathrm{~m} / \mathrm{s}$, calculate the time in hours it would take for the tank to overflow initially if all the pipes A, B and C are opened at the same time.
(3 marks)
18. Mr. Obambla Mbuta a deputy inspector in the police service earns a basic salary of Ksh. 47 000, house allowance of Ksh. 22 000, commuter allowance of Ksh. 8000 and risk allowance of Ksh. 9000 . He has a life insurance policy for which he pays Ksh. 6000 per month and for which he is allowed $15 \%$ as insurance relief. He is also entitled to a personal tax relief of Ksh. 162 per month.

| Monthly income <br> in Kenya shillings | Percentage tax rate <br> in each shilling |
| :---: | :---: |
| $0-10164$ | 10 |
| $10165-19740$ | 15 |
| $19741-29316$ | 20 |
| $29317-38892$ | 25 |
| 38893 and above | 30 |

(a) Using the tax table above:
(i) Determine Mr Kosgei's monthly taxable income.
(ii) Calculate net tax paid by Mr. Obambla Mbuta per month.
(b) Mr Mbuta also had the following monthly deductions from his salary: Sacco loan repayment Ksh. 1 000, WCPS $2 \%$ of basic salary, NHIF Ksh. 1 500. Determine his net monthly salary.
(c) If Mr. Mbuta got an annual increment of $20 \%$ in his basic salary, determine the percentage increase in net tax paid per annum.
19. (a) Complete the table given below by filling in the blank spaces.

| $x$ | 0 | 15 | 30 | 45 | 60 | 75 | 90 | 105 | 120 | 135 | 150 | 165 | 180 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $y=4 \cos 2 x$ | 4.00 |  | 2.00 | 0 | -2.00 |  | -4.00 |  | -2.00 | 0 | 2.00 |  | 4.00 |
| $y=2 \sin (2 x+30)$ | 1.00 | 1.73 | 2.00 | 1.73 |  | 0 | -1.00 |  | -2.00 | -1.73 |  | 0 | 1.00 |

(b) On the grid provided, draw on the same axes, the graph of $y=4 \cos 2 x$ and $y=2 \sin (2 x+30)$ for $0^{\circ} \leq x \leq 180^{\circ}$. Take the scale, 1 cm for $15^{\circ}$ on the $x$ - axis and 1 cm for 1 unit on the $y$-axis.

(c) Use your graph to solve the equation $4 \cos 2 x-2 \sin (2 x+30)=0$
(1 mark)
(d) From the graph state the range of values of $x$ for which $4 \cos 2 x \leq 2 \sin (2 x+30)$.
(1 mark)
20. (a) A wedding committee consisting of three people is to be chosen from five men and seven women.

Draw a tree diagram to represent the above information.

Using the tree diagram above, find the probability that:
(i) All committee members are of the same gender.
(2 marks)
(ii) At least two of the committee members are men.
(b) A tetrahedron is biased such that the probability of a face showing up is given by $\mathrm{P}(\mathrm{t})=\mathrm{mt}$ where m is a constant and $\mathrm{t}=1,2,3$ and 4 (number of the faces). Find the probability that when the tetrahedron is tossed twice the sum of the faces that will show up is 7 .
(3 marks)
21. (a) A carpenter wishes to make a ladder with 15 cross - pieces. The cross - pieces are to diminish uniformly in lengths from 63 cm at the bottom to 28 cm at the top. Calculate:
(i) The length in centimetres of the seventh cross - piece from the bottom.
(ii) The length in centimetres of the fourth cross - piece from the top. (2 marks)
(b) The third, fifth and eighth terms of another Arithmetic Progression (A.P.) form the first three consecutive terms of a Geometric Progression (G.P.). If the common difference of the AP is 3 , find:
(i) The first term of the Geometric Progression.
(ii) The sum of the first eleven terms of the Geometric Progression.
22. The data below shows the marks obtained by 50 students in a certain class.

| Marks | $25-34$ | $35-44$ | $45-54$ | $55-64$ | $65-74$ | $75-84$ | $85-94$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| No. of students | 3 | 6 | 16 | 12 | 8 | 4 | 1 |

(a) Using an assumed mean of 59.5, calculate:
(i) The mean (3 marks)
(ii) The standard deviation of the distribution
(b) Estimate:
(i) The lower quartile of the distribution.
(ii) The pass mark if 34 students passed the exam.
23. (a) The speed $\mathrm{V} \mathrm{m} / \mathrm{s}$ of a moving particle is partly constant and partly varies as time t seconds. It is given that $V=28 \mathrm{~m} / \mathrm{s}$ when $\mathrm{t}=2$ and $\mathrm{V}=53 \mathrm{~m} / \mathrm{s}$ when $\mathrm{t}=7$ seconds. Find the speed of the particle when $t=11$ seconds.
(b) A quantity R varies directly as T and inversely as the cube root of S . Given that $S=64$ when $T=6$ and $R=30$;
(i) Find the formula connecting R, S and T.
(ii) Find the percentage change in R when T is decreased by $10 \%$ and S increased by $25 \%$.
(3 marks)
24. The vertices of a trapezium ABCD ar $\mathrm{A}(1,1), \mathrm{B}(2,2), \mathrm{C}(4,2)$ and $\mathrm{D}(4,1)$ is transformed by the matrix $\mathbf{T}=\left(\begin{array}{cc}1 & -2 \\ 0 & 1\end{array}\right)$ to obtain the image $A^{\prime} \mathrm{B}^{\prime} \mathrm{C}^{\prime} \mathrm{D}^{\prime}$.
(a) Find the coordinates of the image $A^{\prime} B^{\prime} C^{\prime} D^{\prime}$. Hence draw the trapezium and its image on the grid provided below.

(b) Describe the transformation represented by the matrix $\mathbf{T}$.
(c) Draw $\mathrm{A}^{\prime \prime} \mathrm{B}^{\prime \prime} \mathrm{C}^{\prime} \mathrm{D}$ ", the image of the trapezium $\mathrm{A}^{\prime} \mathrm{B}^{\prime} \mathrm{C}^{\prime} \mathrm{D}^{\prime}$ under the transformation represented by the matrix $\mathbf{U}=\left(\begin{array}{cc}0 & -1 \\ -1 & 0\end{array}\right)$.
(d) Find the transformation matrix which maps A"B"C"D" onto ABCD. (2 marks)

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