## Kenya Certificate of Secondary Education

| $121 / 2-\quad$ MATHEMATICS | KASSU JET | Paper 2 |
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|  | June. $2023-2 \frac{1}{2}$ Hours |  |

Name $\qquad$ .Adm.
Serial No
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## Instructions to Candidates

(a)Write your name and Admission number in the spaces provided above.
(b) This paper consists of two sections: Section I and Section $\mathbf{H}$.
(c)Answer ALL questions in section I and ANY five questions in section II.
(d) All answers and workings must be written on the question paper in the spaces provided below each question.
(e)Show all the steps in your calculation, giving your answer at each stage in the spaces below each question.
(f) Non - Programmable silent electronic calculators and KNEC mathematical tables may be used, except where stated otherwise.

## FOR EXAMINER'S USE ONLY

## Section I

| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | TOTA |
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## Section II

| 17 | 18 | 19 | 20 | 21 | 22 | 23 | 24 | TOTAL |
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| Grand Total |  |  |  |  |  |  |  |  |

## SECTION I (50 MARKS)

## Answer all questions in this section

1 A cuboid has a length of 9.75 cm , width of 4.5 cm and exact height of 3.2 cm .
Calculate the relative error in the volume of the cuboid. (3 marks)

2 Find the value of $x$ in the equation.

$$
\log _{3}(3 x-3)-3=2 \log _{3}(x-1)
$$

3. A businessman deposited Ksh. 80,000 ©n a savings account at the beginning of the year, which pays $10.5 \%$ interest per annum compounded quarterly. Find the amount in the account at the beginning of $5^{\text {th }}$ year.
(3 marks)
4. Two quantities $Q$ and $R$ are such that $Q$ varies partly as $R$ and partly varies as the square root of R . Determine the equation connecting Q and R given that $Q=$ 500 when $R=16$ and $Q=800$ when $R=25$.
(3 marks)
5. The position vectors of points $A$ and $B$ are $\boldsymbol{a}=3 \boldsymbol{i}-2 \boldsymbol{j}+4 \boldsymbol{k}$ and $\boldsymbol{b}=-2 \boldsymbol{i}+\boldsymbol{j}$ respectively. A point $R$ divides line $A B$ externally in the ratio $3: 1$. Find the position vector of R in terms of $\boldsymbol{i}, \boldsymbol{j}$ and $\boldsymbol{k}$.
6. (a)Show that the equation $2 \sin x=\frac{4 \cos x-1}{\tan x}$ can be expressed in the form $6 \cos ^{2} x-\cos x-2=0$
(b)Solve the above equation for $-180^{\circ} \leq x \geq 180^{\circ}$
7. Given that $A=\pi(R-r)(R+r)$. Make R the subject of the formula. (3 marks)
8. The following data represents the ages in years at which pupils were admitted into standard four in a local primary school: 12, 10, 9, 11, 13,11. Calculate the standard deviation of their ages.
9. Evaluate : $\int_{0}^{1} \frac{80 x-44 x^{2}-12 x^{3}}{16 x-12 x^{2}} d x$
10. Find the radius and the coordinates of the center of the circle whose equation is $\frac{1}{2} x^{2}+\frac{1}{2} y^{2}=3 x-5 y-9$ and hence draw the circle in the grid below. (4 marks)

11. Simplify $\frac{\sin 45^{\circ}}{1-\cos 30^{\circ}}$ leaving your answer in the form $a \sqrt{b}+\sqrt{c}$ where $\mathrm{a}, \mathrm{b}$ and c are rational numbers.
(3 marks)
12. Solve the simultaneous equations given

$$
\begin{align*}
& x+3 y=10 \\
& x^{2}-x y=8 \tag{4marks}
\end{align*}
$$

13. Expand the expression $\left(x-\frac{1}{2 x}\right)^{6}$ in ascending powers of $x$ and hence state the
14. Construct a tangent through the point $X$ on the circumference of the given circle

15. A tank has two inlet pipes A and B.A fills the tank in three hours while be does so in six hours. Pipe R, the outlet pipe empties a full tank in 4 hrs . The inlet pipes are opened at the same time and left numing for 1.5 hrs . R is then opened and all are left running until the tank is full calculate the total time it takes to fill the tank.
16. Every time a frog jumps forward it jumps half of the previous jump. If the frog initially jumped 20.2 cm calculate the length of the $6^{\text {th }}$ jump and the total distance covered.
(3 marks)

## SECTION II (50 MARKS)

Answer any FIVE questions in this section
17. (a) Complete the table below for the functions of $y=2 \sin \frac{1}{2} x$ and $y=\sin x$ to 2 d.p
(2 marks)

| $x^{0}$ | 0 | 90 | 180 | 270 | 360 | 450 | 540 | 630 | 720 | 810 | 900 |
| :---: | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| $y=2 \sin \frac{1}{2} x$ |  |  |  |  |  |  |  |  |  |  |  |
| $y=\sin x$ |  |  |  |  |  |  |  |  |  |  |  |

(b) On the same axes, draw the graphs of $y=2 \sin \frac{1}{2} x$ and $y=\sin x$ (use 2 units to represent one unit on the $y$ - axis and 1 unit to represent $90^{\circ}$ on the x axis)
(4

(c)Use the graph tog solve the equation $\sin \frac{1}{2} x-\frac{1}{2} \sin x=0$.
(2 marks)
(d)Describe fully the transformation that maps $y=\sin x$ onto $y=2 \sin \frac{1}{2} x$ (2 marks)
18. A and B are two points on the latitude $52^{\circ} N$. The two points lie on the longitudes $30^{\circ} \mathrm{E}$ and $150^{0} \mathrm{~W}$ respectively.
(a)Calculate the:
(i) distance in km from A to B along the parallel of latitude. (Take $\pi=\frac{22}{7}$ and Radius $=6370 \mathrm{~km}$ )
(ii) shortest distance in nautical miles from A to B along the great circle via North pole.
(b) An aircraft took 46 hours to fly frompoint A to B along the parallel latitude.

Given that it took off from A on Monday 11:34am. Calculate:
(i) the speed of aircraft in knots.
(3 marks)
(ii) time and the day of arrival in B. (2 marks)
19. A commercial plane at Wilson Airport is assigned a pilot and a co-pilot for efficient running on daily basis. The pilot must work for more than 2 hours
daily. The hours worked by the co-pilot must be more than one-third the hours worked by the pilot. The total hours worked by both should not be more than 12 hours. The number of hours done by the pilot and twice the number of hours done by the co-pilot should be more than 10 hours. By taking $\boldsymbol{x}$ and $\boldsymbol{y}$ to represent the hours worked by a pilot and a co-pilot respectively
(a)Write down four inequalities to represent the above information.
(b) Use the grid to represent the inequalities in (a) above. (4 marks)

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(c)A pilot is given Ksh. 4500 allowance while a co-pilot pilot is given Ksh.3,200 pocket allowance using a search line or otherwise determine the minimum allowance they earn in a day.
20. Mr. Moneyman earns a basic salary of sh 12560 and house allowance of sh 2,800 per month. Being a civil servant, he is deducted ksh 2640 for National housing which is exempted from taxation. He has also another tax exemption of ksh 360 which is deducted for the National Social Security Fund and he is entitled to a monthly personal relief of sh 1056 .
(a) Calculate his taxable income per annum
(b) The table below shows the tax rates during that year. Use the table to calculate his PAYE.

| Taxable income per year (sh) | Rate (sh for every sh. 20) |
| :---: | :---: |
| $1-72600$ | 2 |
| $72601-145200$ | 3 |
| $145201-217800$ |  |
| $217801-290400$ |  |

(c)The following deductions are also made from his monthly income:

Cooperative shares Ksh. 750.00
Cooperative loan
Ksh. 575.50
Service charge
Ksh. 185.00
Determine Mr. Moneyman's net monthly salary
(2 marks)
21. In the figure below $\mathbf{O P}=\frac{1}{2} \boldsymbol{a}+\boldsymbol{b}, \mathbf{O R}=\frac{7}{2} \boldsymbol{a}-\boldsymbol{b}, \boldsymbol{R} \boldsymbol{Q}=\frac{3}{2} k \boldsymbol{b}+\frac{1}{2} m \boldsymbol{a}$, where $k$ and $m$ are scalars $2 \mathrm{PS}=3 \mathrm{SR}$

(a)Express as simply as possible in terms of $\mathbf{a}$ and $\mathbf{b}$ each of the following vectors.
(i) $\mathbf{P R}$
(ii) PS
(iii) OS
(b) Express $\mathbf{O Q}$ in terms of $\mathbf{a}, \mathbf{b}, k$ and $m$
(c)If Q lies on $\mathbf{O S}$ produced with $\boldsymbol{O Q}: \boldsymbol{O S}=5: 4$, find the value of $k$ and $m$.
(5 marks)

## 22. Draw a line $A B 8$ cm.

a. draw the locus of point T above line AB suchrthat angle $\mathrm{ATB}=90^{\circ}$
(2marks)
b. the locus of C above AB such that triangle $\mathrm{ACB}=9.6 \mathrm{~cm}^{2}$. Label two points M and N in the loci of both T and Csuch that M is nearer to A than B and N is nearer to B than A .
(2marks)
c. find the area enclosed by the locus of T and the locus of C.
d. find the probability that a point chosen at random in the area enclosed by $A B$ and the locus of T is also found in the area enclosed by the locus of T and the locus of C.
23. Two transformations $T_{1}$ and $T_{2}$ are given by matrices $T_{1}=\left(\begin{array}{cc}2 & -1 \\ 1 & 1\end{array}\right)$ and $T_{2}=$ $\left(\begin{array}{cc}1 & 3 \\ -1 & 2\end{array}\right)$. T is a single matrix of transformation equivalent to $T_{1}$ followed by $T_{2}$ a. find $T$.
b. points $\mathrm{A}(-1,1), B(1,2)$ and $C(0,3)$ is mapped onto $A^{\prime} B^{\prime} C^{\prime}$ under T. find the coordinates of $A^{\prime}, B^{\prime}$, and $C^{\prime}$
(2marks)
c. The following figure represents a triangle ABC with vertices $\mathrm{A}(-4,5), \mathrm{B}(-2,2)$ and $C(2,2)$. If the vertex $B(-2,2)$ is mapped onto $B^{\prime}(-2,-2)$ by a shear with the y-axis invariant, draw a triangle $A^{\prime} B^{\prime} C^{\prime}$ the inage of triangle ABC under the shear.

3marks

d. Find the matrix that represent the sheer above

3marks
24. A football match is such that a win garners three points a draw garners one point and a match lost earns no point. The probability of team winning is $40 \%$ lose is $45 \%$ and a draw $15 \%$ if the team plays two games:
a. draw a probability tree diagram to represent all the possible outcomes.

2marks
b. the probability that:
i. they earn six points.
ii. they win at least one match.
(2 marks)
(2 marks)
iii. they will hageat most two points.
(2 marks)
iv. they will garner more than one points.
(2 marks)

