Kenya Certificate of Secondary Education

121/2 - MATHEMATICS - Paper 2
KASSU JET
June. 2023 - $2\frac{1}{2}$ *Hours*

Name………………………………………………………….**Adm**…………..
Serial No……………..**Signature**………….……..**Date**……………………..

Instructions to Candidates

(a)*Write your name and Admission number in the spaces provided above.*
*(b) This paper consists of two sections: **Section** I and **Section II**.*
*(c)Answer **ALL** questions in **section I** and **ANY** five questions in **section II**.*
(d) All answers and workings must be written on the question paper in the spaces provided below each question.
*(e)**Show all the steps in your calculation, giving your answer at each stage in the spaces below each question***.
(f) Non – Programmable silent electronic calculators and KNEC mathematical tables may be used, except where stated otherwise.

FOR EXAMINER'S USE ONLY

Section I

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	TOTA

Section II

17	18	19	20	21	22	23	24	TOTAL

Grand Total

1 A cuboid has a length of 9.75cm, width of 4.5cm and exact height of 3.2cm. Calculate the relative error in the volume of the cuboid. *(3 marks)*

2 Find the value of x in the equation. *(3 marks)*
$$\log_3(3x - 3) - 3 = 2\log_3(x - 1)$$

3. A businessman deposited Ksh. 80,000 in a savings account at the beginning of the year, which pays 10.5% interest per annum compounded quarterly. Find the amount in the account at the beginning of 5th year. *(3 marks)*

4. Two quantities Q and R are such that Q varies partly as R and partly varies as the square root of R. Determine the equation connecting Q and R given that $Q = 500$ when $R = 16$ and $Q = 800$ when $R = 25$. *(3 marks)*

5. The position vectors of points A and B are $\boldsymbol{a} = 3\boldsymbol{i} - 2\boldsymbol{j} + 4\boldsymbol{k}$ and $\boldsymbol{b} = -2\boldsymbol{i} + \boldsymbol{j}$ respectively. A point R divides line AB externally in the ratio 3 : 1. Find the position vector of R in terms of $\boldsymbol{i}, \boldsymbol{j} \ and \ \boldsymbol{k}$. *(4 marks)*

6. (a)Show that the equation $2 \sin x = \frac{4 \cos x - 1}{\tan x}$ can be expressed in the form
$6 \cos^2 x - \cos x - 2 = 0$ *(2 marks)*

(b)Solve the above equation for $-180^0 \leq x \geq 180^0$ *(2 marks)*

7. Given that $A = \pi(R - r)(R + r)$. Make R the subject of the formula. *(3 marks)*

8. The following data represents the ages in years at which pupils were admitted into standard four in a local primary school: 12, 10, 9, 11, 13,11. Calculate the standard deviation of their ages. *(3 marks)*

9. Evaluate : $\int_0^1 \frac{80x - 44x^2 - 12x^3}{16x - 12x^2} \, dx$ *(4 marks)*

10. Find the radius and the coordinates of the center of the circle whose equation is $\frac{1}{2}x^2 + \frac{1}{2}y^2 = 3x - 5y - 9$ and hence draw the circle in the grid below. *(4 marks)*
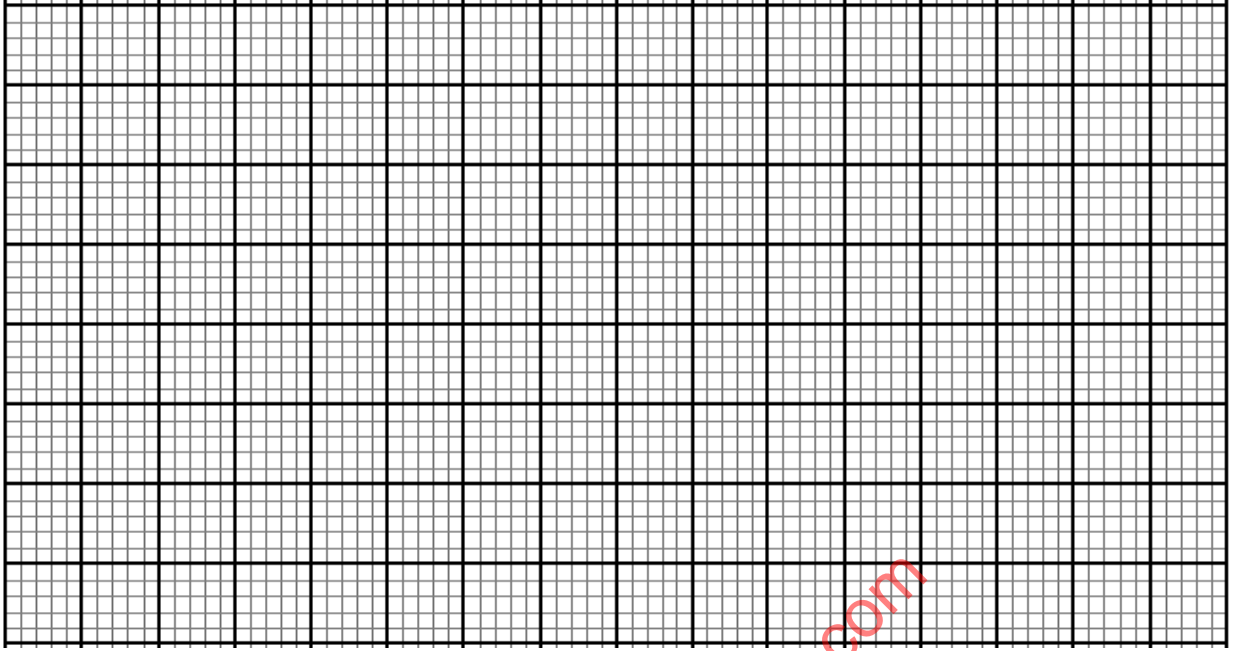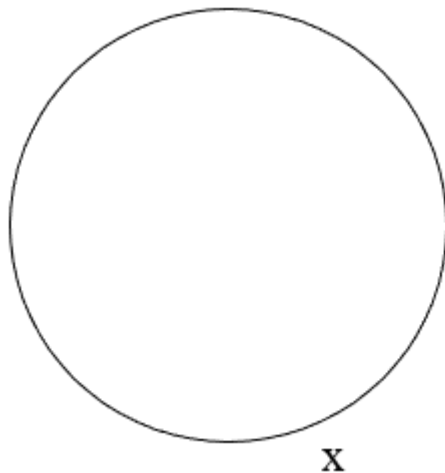
11. Simplify $\frac{\sin 45^0}{1-Cos\ 30^0}$ leaving your answer in the form $a\sqrt{b} + \sqrt{c}$ where a, b and c are rational numbers. *(3 marks)*

12. Solve the simultaneous equations given $\begin{array}{l} x + 3y = 10 \\ x^2 - xy = 8 \end{array}$ *(4 marks)*

13. Expand the expression $\left(x - \frac{1}{2x}\right)^6$ in ascending powers of x and hence state the constant term. *(2 marks)*

14. Construct a tangent through the point X on the circumference of the given circle

X

15. A tank has two inlet pipes A and B. A fills the tank in three hours while be does so in six hours. Pipe R, the outlet pipe empties a full tank in 4hrs. The inlet pipes are opened at the same time and left running for 1.5 hrs. R is then opened and all are left running until the tank is full calculate the total time it takes to fill the tank.

(4 marks)

16. Every time a frog jumps forward it jumps half of the previous jump. If the frog initially jumped 20.2 cm calculate the length of the 6th jump and the total distance covered. *(3 marks)*
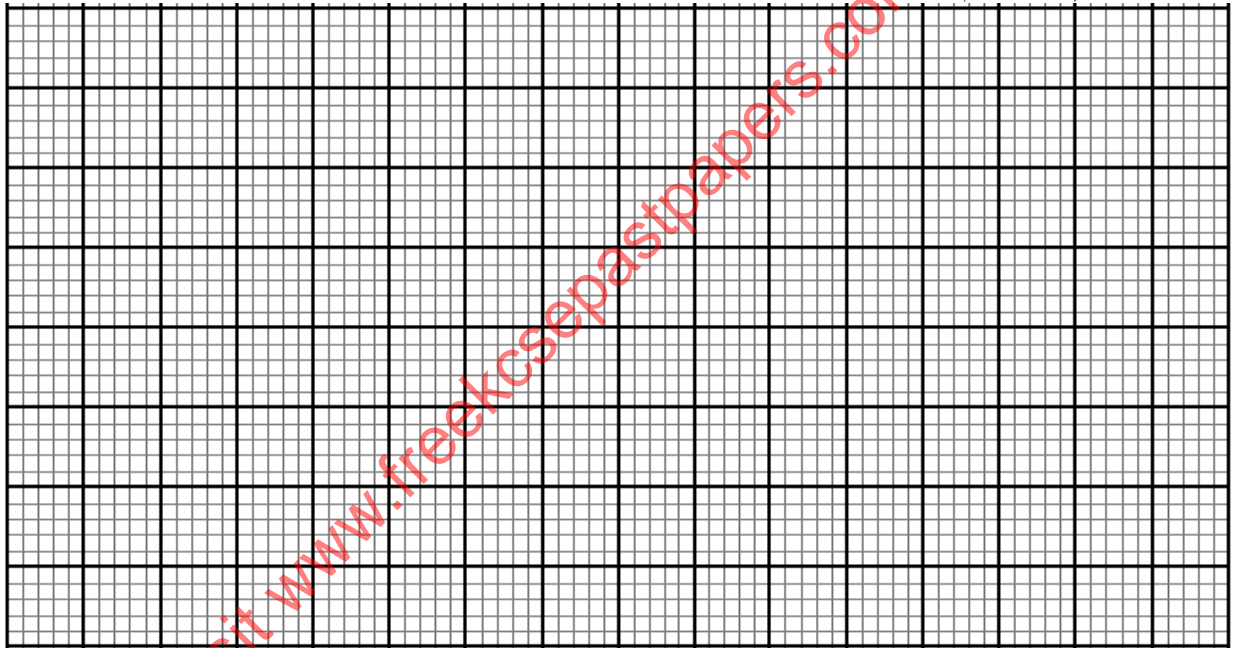
*Answer any **FIVE** questions in this section*

17. (a) Complete the table below for the functions of $y = 2\sin\frac{1}{2}x$ and $y = \sin x$ to
2 d.p *(2 marks)*

x^0	0	90	180	270	360	450	540	630	720	810	900
$y = 2\sin\frac{1}{2}x$											
$y = \sin x$											

(b) On the same axes, draw the graphs of $y = 2\sin\frac{1}{2}x$ and $y = \sin x$ (use 2 units
to represent one unit on the y- axis and 1 unit to represent 90^0 on the x axis)
(4 marks)

(c) Use the graph to solve the equation $\sin\frac{1}{2}x - \frac{1}{2}\sin x = 0$. *(2 marks)*

(d) Describe fully the transformation that maps $y = \sin x$ onto $y = 2\sin\frac{1}{2}x$
(2 marks)

18. A and B are two points on the latitude 52^0N. The two points lie on the longitudes 30^0E and 150^0W respectively.

(a)Calculate the:

(i) distance in km from A to B along the parallel of latitude. (Take $\pi = \frac{22}{7}$ and $Radius = 6370\ km$) *(3 marks)*

(ii) shortest distance in nautical miles from A to B along the great circle via North pole. *(2 marks)*

(b) An aircraft took 46 hours to fly from point A to B along the parallel latitude. Given that it took off from A on Monday 11:34am. Calculate:

(i) the speed of aircraft in knots: *(3 marks)*

(ii) time and the day of arrival in B. *(2 marks)*

19. A commercial plane at Wilson Airport is assigned a pilot and a co-pilot for efficient running on daily basis. The pilot must work for more than 2 hours daily. The hours worked by the co-pilot must be more than one-third the hours worked by the pilot. The total hours worked by both should not be more than 12 hours. The number of hours done by the pilot and twice the number of hours done by the co-pilot should be more than 10 hours. By taking x and y to represent the hours worked by a pilot and a co-pilot respectively

(a) Write down four inequalities to represent the above information.   *(4 marks)*

(b) Use the grid to represent the inequalities in (a) above.   *(4 marks)*
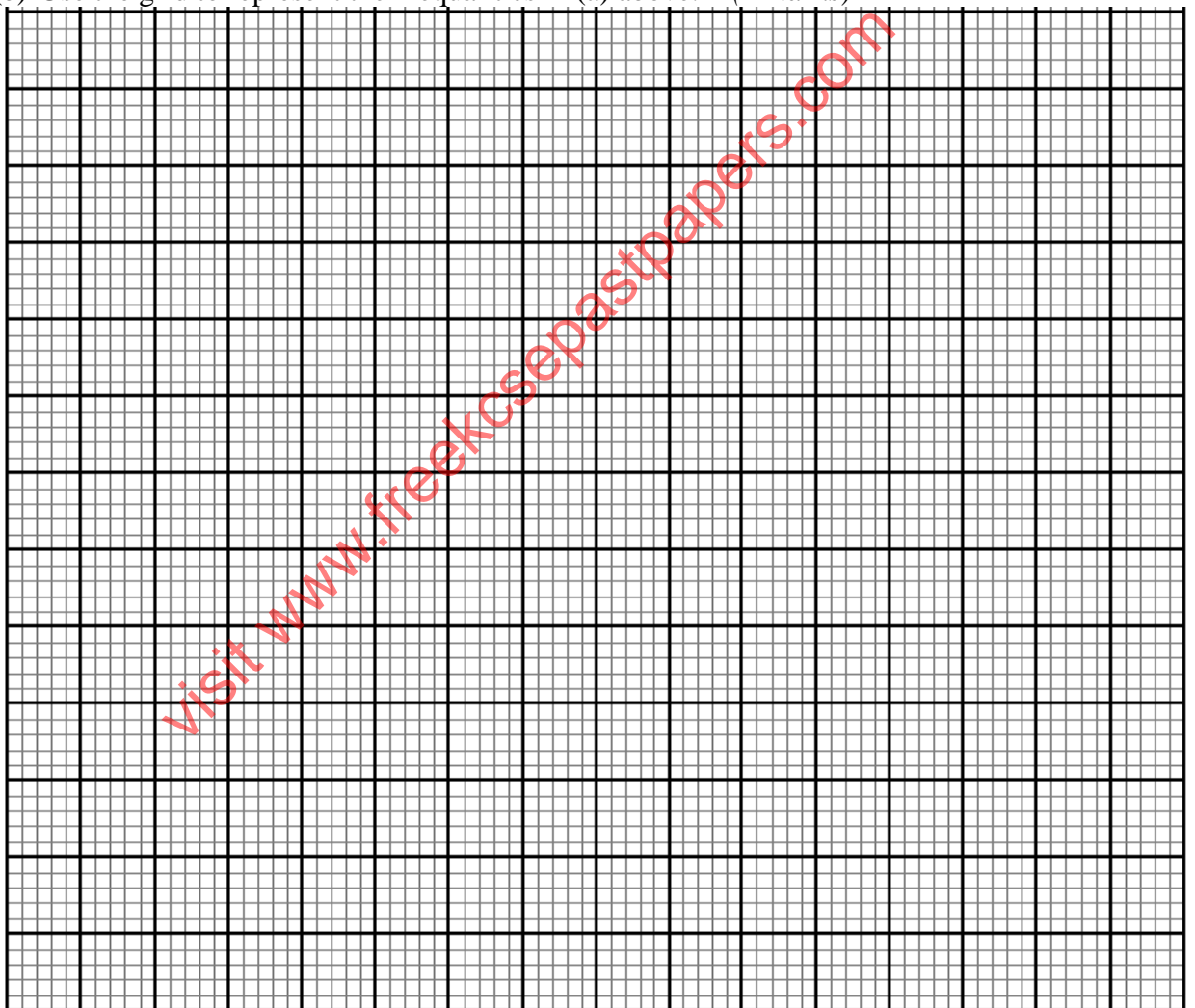
(c) A pilot is given Ksh.4500 allowance while a co-pilot pilot is given Ksh.3,200 pocket allowance using a search line or otherwise determine the minimum allowance they earn in a day.   *(2 marks)*

20. Mr. Moneyman earns a basic salary of sh 12560 and house allowance of sh 2,800 per month. Being a civil servant, he is deducted ksh 2640 for National housing which is exempted from taxation. He has also another tax exemption of ksh 360 which is deducted for the National Social Security Fund and he is entitled to a monthly personal relief of sh 1056.

(a) Calculate his taxable income per annum *(3 marks)*

(b) The table below shows the tax rates during that year. Use the table to calculate his PAYE. *(5 marks)*

Taxable income per year (sh)	Rate (sh for every sh. 20)
1 – 72600	2
72601 – 145 200	3
145 201 – 217 800	5
217 801 – 290 400	7

(c)The following deductions are also made from his monthly income:

Cooperative shares   Ksh. 750.00
Cooperative loan   Ksh. 575.50
Service charge   Ksh. 185.00
Determine Mr. Moneyman's net monthly salary *(2 marks)*

21. In the figure below $\mathbf{OP} = \frac{1}{2}\boldsymbol{a} + \boldsymbol{b}, \boldsymbol{OR} = \frac{7}{2}\boldsymbol{a} - \boldsymbol{b}, \boldsymbol{RQ} = \frac{3}{2}k\boldsymbol{b} + \frac{1}{2}m\boldsymbol{a},$ where k and m are scalars $2\text{PS} = 3\text{SR}$
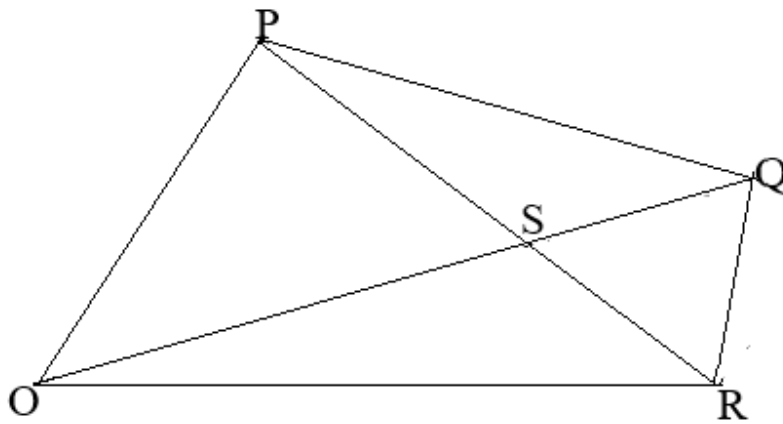
(a) Express as simply as possible in terms of **a** and **b** each of the following vectors.

(i) **PR**   (1 mk)

(ii)   **PS**   (1 mk)

(iii)   **OS**   (1 mark)

(b)   Express **OQ** in terms of **a, b,** k and m (2 marks)

(c) If Q lies on **OS** produced with $\boldsymbol{OQ}:\boldsymbol{OS} = 5 : 4$, find the value of k and m.

(5 marks)

22. Draw a line AB 8 cm. (1 mark)

a. draw the locus of point T above line AB such that angle ATB $=90^0$
(2marks)

b. the locus of C above AB such that triangle ACB = 9.6 cm². Label two points M and N in the loci of both T and C such that M is nearer to A than B and N is nearer to B than A. (2marks)

c. find the area enclosed by the locus of T and the locus of C. (3 marks)

d. find the probability that a point chosen at random in the area enclosed by AB and the locus of T is also found in the area enclosed by the locus of T and the locus of C. (2marks)

23. Two transformations T_1 and T_2 are given by matrices $T_1 = \begin{pmatrix} 2 & -1 \\ 1 & 1 \end{pmatrix}$ and $T_2 = \begin{pmatrix} 1 & 3 \\ -1 & 2 \end{pmatrix}$. T is a single matrix of transformation equivalent to T_1 followed by T_2

   a. find T.   (2marks)

b. points $A(-1, 1)$, $B(1, 2)$ and $C(0, 3)$ is mapped onto $A'B'C'$ under T. find the coordinates of A', B', and C' (2marks)

c. The following figure represents a triangle ABC with vertices A(-4, 5), B(-2, 2) and C(2, 2). If the vertex B(-2, 2) is mapped onto B'(-2, -2) by a shear with the y-axis invariant, draw a triangle $A'B'C'$ the image of triangle ABC under the shear.

                                                3marks
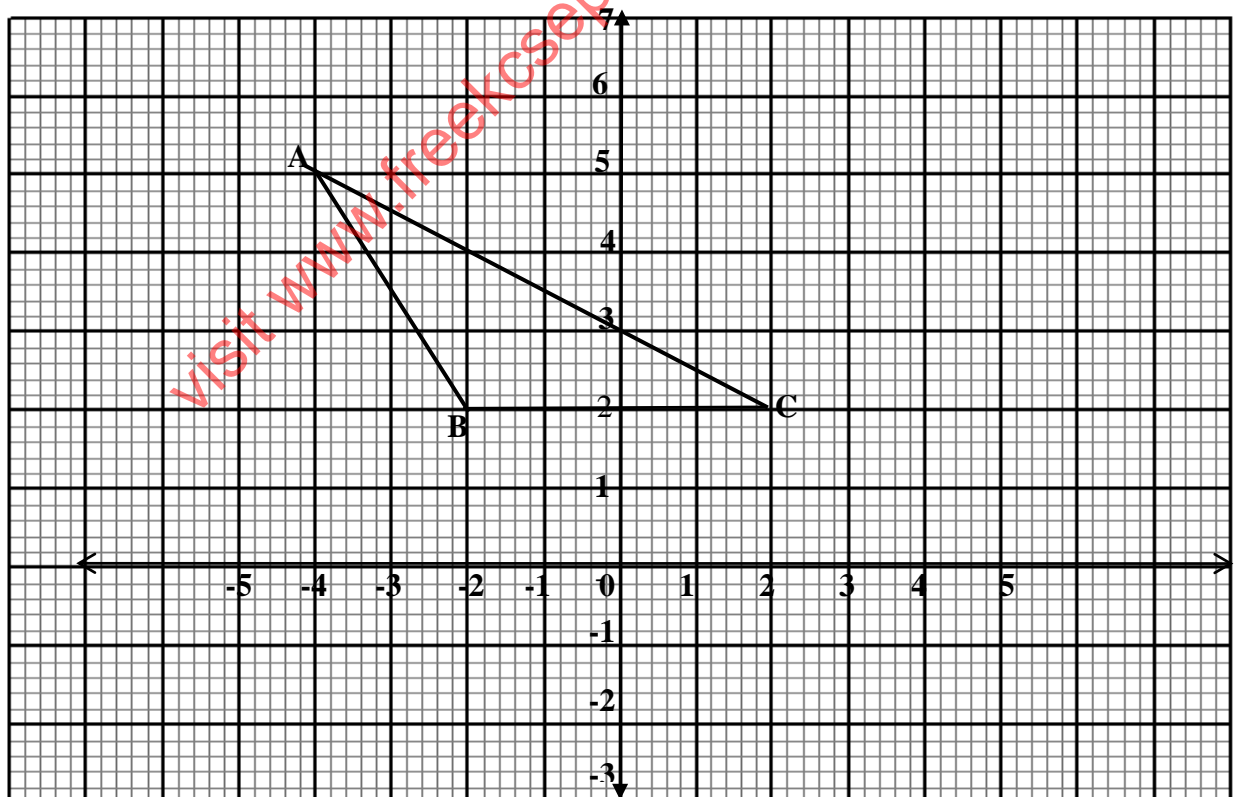
d. Find the matrix that represent the sheer above   3marks

24. A football match is such that a win garners three points a draw garners one point and a match lost earns no point. The probability of team winning is 40% lose is 45% and a draw 15% if the team plays two games:

   a. draw a probability tree diagram to represent all the possible outcomes.

2marks

   b. the probability that:

i. they earn six points. (2 marks)

ii. they win at least one match. (2 marks)

iii. they will have at most two points. (2 marks)

iv. they will garner more than one points. (2 marks)